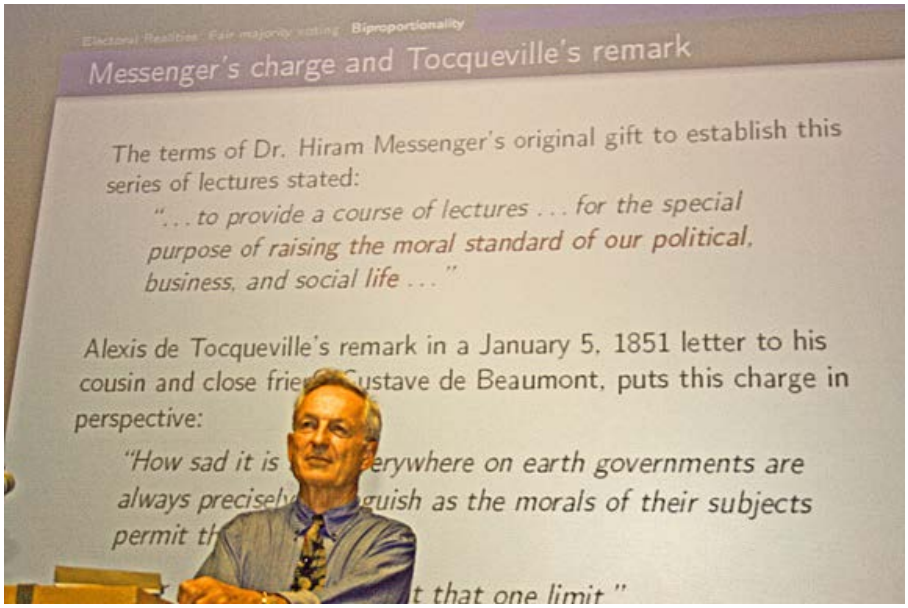
Messenger Lectures

Cornell University

September 25--27, 2007



Michel Balinski, Professor Emeritus, École Polytechnique, Paris, presented a series of three Messenger Lectures entitled:

Representing, Electing and Ranking

The abstracts and references for these lectures can be found here.
The slides and streaming video of the individual lectures are linked below.

Lecture 1: "How to apportion fairly: Aristotle, the Talmud, the U.S. Congress and kidney transplants" slides video
Tuesday, September 25, 2007, 4:45 p.m., Schwartz Auditorium, Rockefeller Hall

Lecture 2: "How to elect and to rank: Overcoming the paradoxes of social choice, electing a President of France and ranking wines" slides video

Wednesday, September 26, 2007, 7:30 p.m., Schwartz Auditorium, Rockefeller Hall

Lecture 3: "How to eliminate gerrymandering: A new approach to representation, its realization in Zürich and its application to the U.S. Congress" slides video

Thursday, September 27, 2007, 4:45 p.m., Schwartz Auditorium, Rockefeller Hall

Biographical information on M.L. Balinski

Representing, Electing and Ranking

Michel Balinski, École Polytechnique, Paris

Mechanisms -- "instruments or processes, physical or mental, by which something is done or comes into being" -- have been invented across the ages to regulate human activities of all kinds: markets and their prices for economic decisions, electoral systems for political decisions, ad hoc rules of every imaginable variety for competitions (sports, musical, wine, beauty or other, or for whom is to have the priority for a kidney transplant).

What mechanisms should be used? The answers depend on the contexts and the mathematical properties they elicit. Practice demands that they be simple, clear and understood by all: else they would be suspect, hence rejected. The focus here will be on the mechanisms for elections (with asides concerning their applications elsewhere).

Lecture 1. **How to apportion fairly:** Aristotle, the Talmud, the U.S Congress and kidney transplants.

The story begins with Aristotle and the Talmud, and goes on to the disputes over the meaning of "Representatives ... shall be apportioned among the several States ...according to their respective numbers ..." among Washington, Hamilton, Jefferson, J.Q. Adams, Webster and later Morse, von Neumann and Cornell's Willcox.

Underlying concepts of fairness determine the mechanism that should be used to apportion representatives. They show why Daniel Webster and after him Walter Willcox had the right idea and the phalanx of the American mathematical elite of the day -- which successfully argued for the method used since 1940 -- erred. It also explains why mechanisms for allocating kidneys to awaiting patients can be unfair.

Lecture 2. **How to elect and to rank:** overcoming the paradoxes of social choice, electing a President of France and ranking wines.

Ramon Llull in 1299 and Nicolas Cusanus in 1433 proposed mechanisms for electing and ranking that continue to be championed to the present day, though they are attributed, respectively, to the Marquis de Condorcet (1789), and the Chevalier de Borda (1770). They established a way of seeing the problem and modelled it in terms that remains the central paradigm of the theory of social choice to this day. As a consequence, Arrow's "impossibility theorem" showing that Condorcet's famous paradox cannot be escaped, has been interpreted as meaning that there is no satisfactory mechanism for either designating a winner or determining a ranking among competitors.

A new theory -- whose antecedents may be traced to the Marquis de Laplace and Sir Francis Galton -- sees the problem and models it differently. It suggests that the usual interpretation is wrong: Arrow's theorem means that there is no satisfactory mechanism unless there is a common language of measuring the qualities of candidates (or wines) or the performances of competitors. But when a common language exists -- and practice shows they do exist -- then one mechanism should be used: the majority judgement. This is explained in the context of the 2007 French presidential elections (and, time permitting, in the context of judging wines).

Lecture 3. **How to eliminate gerrymandering:** a new approach to representation, its realization in Zürich and its application to the U.S. Congress.

Elbridge Gerry -- Governor of Massachusetts (1810-12), Vice President of the United States (1813-14) -- gave his name to the dubious "practice of dividing a geographical area into electoral districts, often of highly irregular shape, to give one political party an unfair advantage." This ancient fine art has become a new 21st century technology that, some say, determines electoral outcomes more than do the voters. And yet it redistricts to meet the one criterion championed by the Supreme Court -- "a good-faith effort to achieve precise mathematical equality": every one of Pennsylvania's 19 districts has a census population of either 646,371 or 646,372, and every one of Texas' 32 has a population of 651,619 or 651,620. Justice John Harlan was unusually prescient in his dissenting opinion of 1969 (Wells v. Rockefeller 394 U.S. 542): "The fact of the matter is that the rule of absolute equality is perfectly compatible with 'gerrymandering' of the

worst sort. A computer may grind out district lines which can totally frustrate the popular will ... The legislature must do more than satisfy one man, one vote; it must create a structure which will in fact as well as theory be responsive to the sentiments of the community ... Even more than in the past, district lines are likely to be drawn to maximize the political advantage of the party temporarily dominant in public affairs."

A new mechanism (extending that of lecture 1) provides the structure sought by John Harlan. It is today the law of the land for the legislative elections of the city and the canton of Zürich, due to a suit of a citizen who complained that his constitutional right to an equal vote was violated by the old system. Adopted for U.S. congressional elections, that mechanism would eliminate the possibility of gerrymandering for political advantage.

References:

Lecture 1

1. Michel L. Balinski and H. Peyton Young, *Fair Representation: Meeting the Ideal of One Man, One Vote*, Yale University Press, New Haven CT and London, 1982; 2nd edition, Brookings Institution Press, Washington DC, 2001.

2. Michel Balinski, “What is just?” *American Mathematical Monthly* 112 (2005) 502-511.

Lecture 2

1. Michel Balinski and Rida Laraki, “A theory of measuring, electing and ranking,” *Proceedings of the National Academy of Sciences, USA* 104 (2007) 8720-8725.

2. Michel Balinski and Rida Laraki, “Le jugement majoritaire : l’expérience d’Orsay,” *Commentaire* N° 118, été 2007, 413-419.

3. Michel Balinski and Rida Laraki, *One-Value, One-Vote: Measuring, Electing and Ranking*, forthcoming.

4. See also the web site: www.ceco.polytechnique.fr/jugement-majoritaire.html

Lecture 3

1. Michel Balinski, Le suffrage universel inachevé, Belin Paris, 2004.

2. Michel Balinski, “Apportionment : uni- and bi-dimensional,” in B. Simeone et F. Pukelsheim (Eds.), *Mathematics and Democracy. Recent Advances in Voting Systems and Collective Choice*, Springer, Berlin and Heidelberg, 2006, 43-53.

3. Michel Balinski, “Fair majority voting (or how to eliminate gerrymandering),” to appear in *American Mathematical Monthly*, February 2008.

4. Michel Balinski and Friedrich Pukelsheim, “Die Mathematik der doppelten Gerechtigkeit,” *Spektrum der Wissenschaft*, April 2007, 76-80.

5. Michel Balinski and Friedrich Pukelsheim, “Matrices and politics,” in E. P. Liski, J. Isotalo, S. Puntanen and G. P. H. Styan (Eds.), *Festschrift for Tarmo Pukkila*, Department of Mathematics, Statistics and Philosophy, University of Tampere, Finland, 2006, 233-242.

HOW TO APPORTION FAIRLY

Part 1: Representing, electing and ranking

Michel Balinski
École Polytéchnique and CNRS

Cornell University
September 25, 2007

REPRESENTING, ELECTING, and RANKING

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Lecture 2: Why the "first-past-the-post" method of electing one among several candidates—the most used method of all—is seriously defective, often elects the "wrong" candidate . . . *and what to do about it*.

Lecture 3: Why blatant political gerrymandering is unavoidable in today's system . . . *and what to do about it*.

The sorry state of representation in the United States

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- 51 Democratic Senators represent 58% of the population,
- 49 Republican Senators represent 42% of the population.

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- 50 Senators represent 16% of the population,
- 50 Senators represent 84% of the population.
- 51 Democratic Senators represent 58% of the population,
- 49 Republican Senators represent 42% of the population.
- In fact, as I will argue, a minority of voters can elect a majority of the U.S. House of Representatives (and probably has).

The sorry state of representation elsewhere

In the United Kingdom

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In the United Kingdom

The "winners" of the last six British elections:

	1983	1987	1992	1997	2001	2005
Votes	42.4%	42.2%	41.9%	43.2%	40.7%	35.2%
Seats	61.1%	57.8%	51.6%	63.4%	62.5%	55.1%

The sorry state of representation elsewhere

In the United Kingdom

The "winners" of the last six British elections:

	1983	1987	1992	1997	2001	2005
Votes	42.4%	42.2%	41.9%	43.2%	40.7%	35.2%
Seats	61.1%	57.8%	51.6%	63.4%	62.5%	55.1%

2005 election:

	Votes	Seats
Labour	35.2%	55.1%
Conservatives	32.3%	30.7%
Liberals	22.0%	9.6%

The sorry state of representation elsewhere

In France

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but today's Assemblée Nationale districts drawn in 1986 on the basis of the census of 1982. By the last available data (based on 1999 census) populations of districts are:

2^{nd} Lozère	2^{nd} Val d'Oise	1^{st} Var	6^{th} Var
34,374	188,200	73,946	180,153

Contents

1 First Presidential Veto

2 Why Proportionality?

3 Fair Representation

4 Allocating Kidneys

United States Constitution

Article I, section 2:

> *Representatives and direct Taxes shall be apportioned among the several States . . . according to their respective Numbers . . . The actual Enumeration shall be made . . . every subsequent Term of ten years . . . The Number of Representatives shall not exceed one for every thirty thousand, but each State shall have at least one Representative.*

1791-1792: House proposal, 112 seats, Jefferson's method

State	Population	$\div 30,000$	Bill	
Virginia	630,560	21.019	21	
Massachusetts	475,327	15.844	15	
Pennsylvania	432,879	14.429	14	
North Carolina	353,523	11.784	11	
New York	331,589	11.053	11	
Maryland	278,514	9.284	9	
Connecticut	236,841	7.895	7	
South Carolina	206,236	6.875	6	
New Jersey	179,570	5.986	5	
New Hampshire	141,822	4.727	4	
Vermont	85,533	2.851	2	
Georgia	70,835	2.361	2	
Kentucky	68,705	2.290	2	
Rhode Island	68,446	2.282	2	
Delaware	55,540	1.851	1	

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State	Population	$\div 30,000$	Bill	Quota of 112
Virginia	630,560	21.019	21	**19.531**
Massachusetts	475,327	15.844	15	14.723
Pennsylvania	432,879	14.429	14	13.408
North Carolina	353,523	11.784	11	10.950
New York	331,589	11.053	11	**10.271**
Maryland	278,514	9.284	9	8.627
Connecticut	236,841	7.895	7	7.336
South Carolina	206,236	6.875	6	6.388
New Jersey	179,570	5.986	5	5.562
New Hampshire	141,822	4.727	4	4.393
Vermont	85,533	2.851	2	2.649
Georgia	70,835	2.361	2	2.194
Kentucky	68,705	2.290	2	2.128
Rhode Island	68,446	2.282	2	2.120
Delaware	55,540	1.851	1	**1.720**

1791-1792: Hamilton's retort: Congress's bill, 120 seats

State	Quota of 120	Bill
Virginia	20.926*	21
Massachusetts	15.774*	16
Pennsylvania	14.366	14
North Carolina	11.732*	12
New York	11.004	11
Maryland	9.243	9
Connecticut	7.860*	8
South Carolina	6.844*	7
New Jersey	5.959*	6
New Hampshire	4.707*	5
Vermont	2.839*	3
GA	2.351	2
Kentucky	2.280	2
Rhode Island	2.271	2
Delaware	1.843*	2

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(2) *assign 9 left-over seats to largest remainders* (*).

1791-1792: the Virginians' reaction

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James Madison (letter to his father):

"*The secret of the business is that by these different rules the relative number of East.n & South.n members is varied. The number 120 is made out by applying 1 for 30,000 . . . and allowing to* ***fractions*** *of certain amount an additional member.*"

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Richard Henry Lee (letter to *his* father):
"*Six Eastern States have one apiece more than they ought, Jersey and Delaware the same,'. . . if the plain constitutional mode had been pursued of dividing the number of people in* ***each State Respectively*** *by the agreed ration of 30,000. But by a certain arithmetico-political sophistry an arrangement of six to two against the South has been made . . .*"

1791-1792: Hamilton's method

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I. *The aggregate numbers of the United States, are divided by 30,000 which gives the total number of representatives, or 120.*

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"The following process has been pursued by [the Act]:

I. *The aggregate numbers of the United States, are divided by 30,000 which gives the total number of representatives, or 120.*

II. *This number is apportioned among the several states by the following rule—As the aggregate numbers of the* ***United States*** *are to the* ***total number*** *of representatives found as above, so are the* ***particular numbers*** *of* ***each state*** *to the numbers of each state to the number of representatives of such state.* But

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III. *As this second process leaves a residue of Eight out of the 120 members unapportioned, these are distributed among those states which upon that second process have the largest fractions or remainders."*

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I answer, then, that taxes must be divided ***exactly*** *and representatives* ***as nearly*** *is the* ***nearest***, ***ratio*** *will admit; and that fractions must be neglected because the Constitution . . . has left them unprovided for."*

1791-1792: Jefferson's method

"The bill does not say that it has given the residuary representatives ***to the greatest fractions****; though in fact it has done so. It seems to have avoided establishing that into a rule, lest it might not suit on another occasion. Perhaps it may be found the next time more convenient to distribute them* ***among the smaller States****; at another time* ***among the larger States****; at other times according to any other crochet which ingenuity may invent, and the combinations of the day give strength to carry; or they may do it arbitrarily by open bargains and cabals.*

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120 being once found . . . We must take the nearest common divisor, . . . that divisor which applied to every State, gives to them such numbers as, added together, come nearest to 120."

1791-1792: the Virginians' decision

Jefferson's account of April 5:

"[Washington] observed that the vote for & against the bill was perfectly geographical, a northern agt a southern vote, & he feared he should be thought to be taking side with a southern party. I admitted this motive of delicacy, but that it would not induce him to do wrong . . . He here expressed his fear that there would ere long, be a separation of the Union . . . He went home, sent for Randolph . . . desired him to get Mr. Madison . . . They came. Our minds had been before made up. We drew the instrument."

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James Madison and Edmond Randolph, attorney general, were, of course, fellow Virginians.

1791-1792: the first presidential veto of US history

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Jefferson recorded in his memoirs:

"A few of the hottest friends of the bill expressed passion, but the majority was satisfied, and both in and out of doors it gave pleasure to have, at length, an instance of the negative being exercised."

1791-1792: the outcome, Jefferson's method

State	Quota of 105	Bill: Jefferson	
Virginia	18.310	19	
Massachusetts	13.803	14	
Pennsylvania	12.570	13	
North Carolina	10.266	10	
New York	9.629	10	
Maryland	8.088	8	
Connecticut	6.877	7	
South Carolina	5.989	6	
New Jersey	5.214	5	
New Hampshire	4.118	4	
Vermont	2.484	2	
Georgia	2.057	2	
Kentucky	1.995	2	
Rhode Island	1.988	2	
Delaware	1.613	1	

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- The presidential election of 1876:

Candidate	Popular vote	Electoral College
Rutherford B. Hayes	4,036,298	185
Samuel J. Tilden	4,300,590	184

Had the apportionment been fair—had Daniel Webster's method been used as Walter Willcox urged—1 electoral vote would have been shifted from Hayes to Tilden.

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- Hill's method—still used!—definitely replaced Webster's in 1940: 1 seat shifted (Arkansas was safely Democratic):

State	Population	Quota	Hill	Webster
Arkansas	1,949,387	6.473	7	6
Michigan	5,256,106	17.453	17	18

The problem

Who was right: Jefferson or Hamilton?

The problem

Who was right: Jefferson or Hamilton?

Or someone else?

The problem

Who was right: Jefferson or Hamilton?

Or someone else?

Why?

Contents

1 First Presidential Veto

2 Why Proportionality?

3 Fair Representation

4 Allocating Kidneys

Proportionality

Aristotle's eloquence triumphs despite its tautological aspects:

"This, then is what the just is—the proportional; the unjust is what violates proportion . . . [The] justice which distributes common possessions is always in accordance with the kind of proportion mentioned above; . . . and the injustice opposed to this kind of justice is that which violates proportion."

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"The just . . . is a species of the proportionate . . . For proportion is equality of ratios, and involves four terms at least. . . ; and the ratio between one pair is the same as that between another pair; for there is a similar distinction between the persons and the things. As the term A, then, is to B, so will C be to D, and therefore, alternatively, as A is to C, B will be to D . . ."

Why proportionality?

Why is it a well nigh universal belief that when something is to be shared, what is fair is what is proportional?

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Is it merely what Blaise Pascal asserts?:

"Custom makes equity for the sole reason that it is received; it is the mysterious foundation of its authority."

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No! A much more fundamental principle lurks in the background.

Why proportionality?

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That is why proportionality seems fair!

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Two hold a garment. One of them says, "I found it" and the other says, "I found it." One of them says, "It is all mine" and the other says, "It is all mine." Then the one shall swear that his share in it is not less than half, and the other shall swear that his share is not less than half, and [it] shall then be divided between them.

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If one says, "It is all mine," and the other says, "Half of it is mine," he who says "It is all mine," shall swear that his share in it is not less than three-quarters, and he who says, "Half of it is mine," shall swear that his share in it is not less than a quarter. The former then receives three-quarter and the latter receives one-quarter.

The contested garment

Claimants	Claims		CG rule	Proportional rule
A	1		3/4	2/3
B	1/2		1/4	1/3
	Total claim: $1\frac{1}{2}$	Estate:	1	1

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Explanation 1: B concedes $\frac{1}{2}$ to A, what remains is shared equally. *I.e.*, A receives $\frac{1}{2} + \frac{1}{2}(\frac{1}{2}) = \frac{3}{4}$ and B receives $\frac{1}{2}(\frac{1}{2}) = \frac{1}{4}$.

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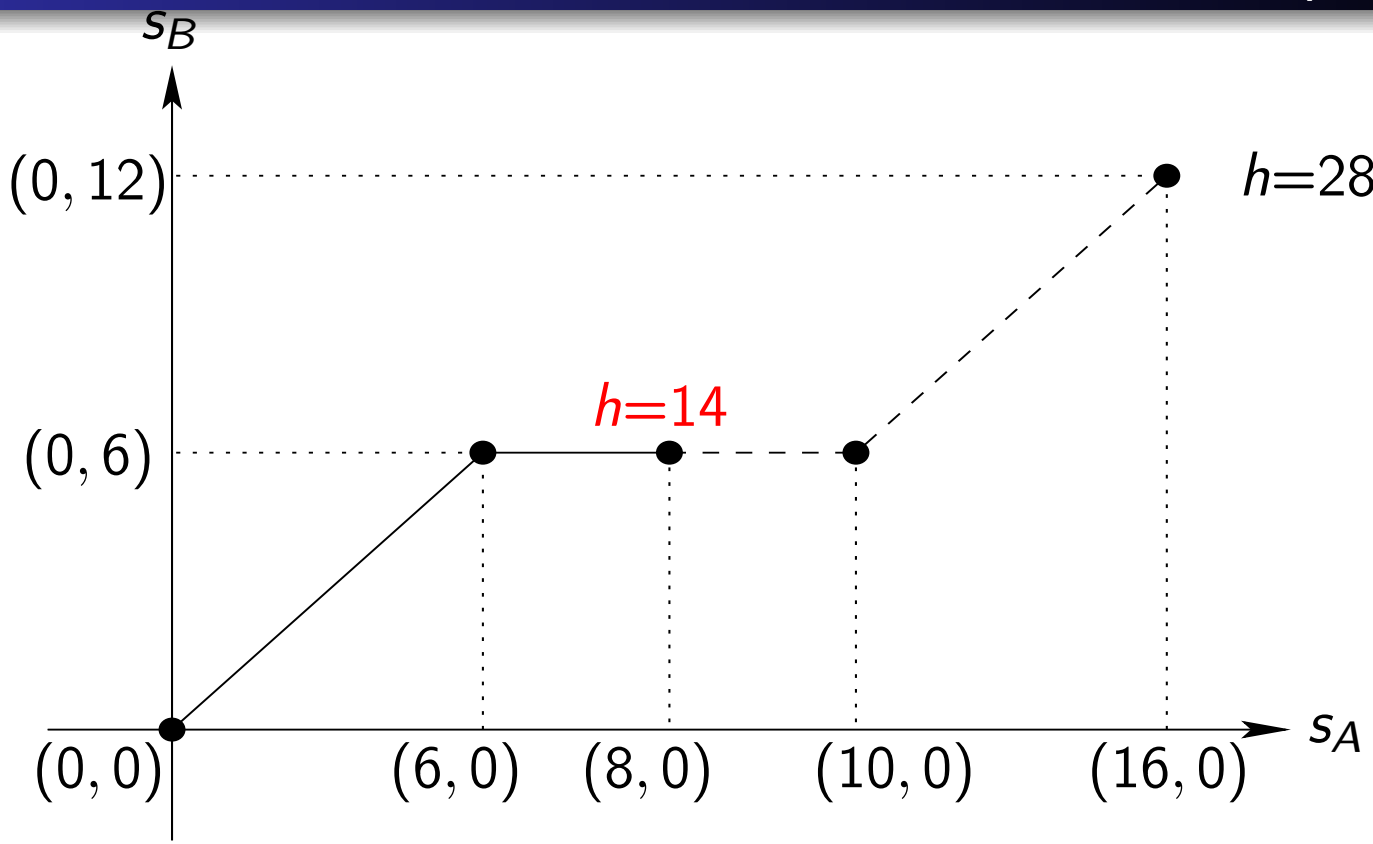
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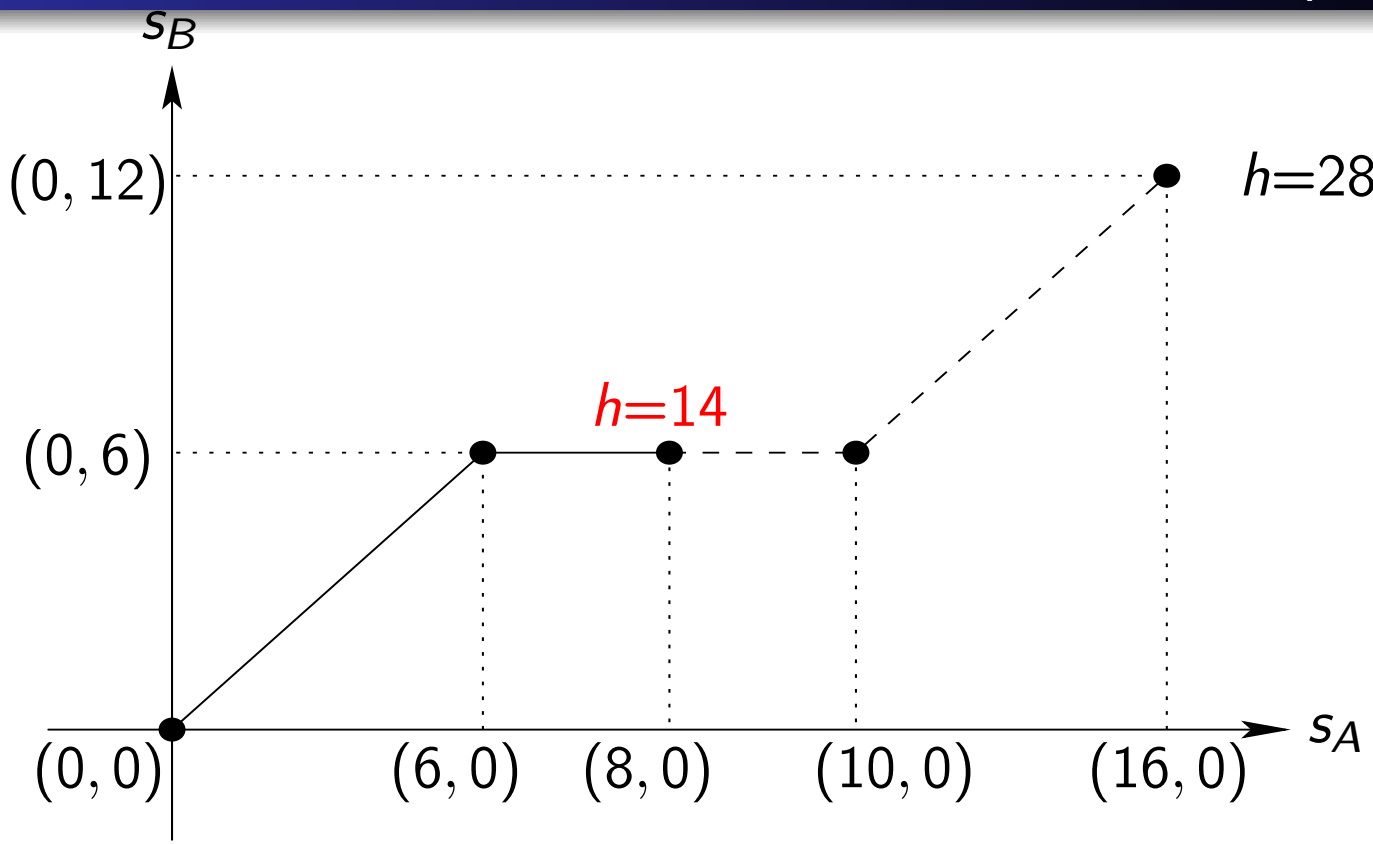
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Explanation 2: Equal losses. *I.e.*, A loses $1 - \frac{3}{4} = \frac{1}{4}$ and B loses $\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$.

The contested garment: all allocations for claims (16, 12)



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For estates that are half the total claim, $0 \leq h \leq 14$:

$$(s_A, s_B) = (\min\{\lambda, 8\}, \min\{\lambda, 6\}),$$

where λ is chosen so that $s_A + s_B = h$.

The contested garment rule: all allocations for claims $(16, 12)$

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For estates h that are more than half the total claim, $14 \leq h \leq 28$:

Calculate the losses for $28 - h$, subtract them from the claims.

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So if the estate is $h = 10$, then $\lambda = 5$ and

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If $h = 18 = 28 - 10$, then

$$(s_A, s_B) = (16 - 5, 12 - 5) = (11, 7).$$

The marriage contract

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"If a man who was married to three wives died, and the Kethubah was a maneh [100 zuz], of the other two hundred zuz, and of the third three hundred zuz and the estate [was worth a hundred zuz, the sum] is divided equally.

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"If a man who was married to three wives died, and the Kethubah was a maneh [100 zuz], of the other two hundred zuz, and of the third three hundred zuz and the estate [was worth a hundred zuz, the sum] is divided equally.

If the estate [was worth] two hundred zuz [the claimant] of the maneh [100 zuz] receives fifty zuz and [the other claimants receive each] three gold denarii [75 zuz].

The marriage contract

If the estate [was worth] three hundred zuz, [the claimant] of the maneh [100 zuz] receives fifty zuz and the [the claimant] of two hundred zuz [receives] a maneh [100 zuz] while [the claimant] of the three hundred zuz [receives] six gold denarii [150 zuz]. Similarly, if three persons contributed to a joint fund and they had made a loss or a profit they share in the same manner."

The marriage contract

Claimants	Claims		Case 1	Case 2	Case 3
A	100		$33\frac{1}{3}$	50	50
B	200		$33\frac{1}{3}$	75	100
C	300		$33\frac{1}{3}$	75	150
	Total claim: 600	Estate:	100	200	300

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Case 3: when the estate is worth 300, proportional division,

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Case 2: when the estate is worth 200, . . . why these shares?

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Case 1: when the estate is worth 100, equal division,

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What **rule of fair apportionment** did the *Kethuboth* have in mind? This question was not answered until 1985 (*via* very sophisticated concepts of game theory).

The *Kethuboth* rule

A very simple observation gives the answer: coherence.

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Specifically, **coherence with the rule of contested garment:**

Every pair of claimants must share what they receive together in accordance with the rule of the contested garment.

Case 1: each pair receives $66\frac{2}{3}$ together, each claimant claims all, so the CG-rule divides the amount equally.

Claimants	Claims	*Kethuboth*
A	100	$33\frac{1}{3}$
B	200	$33\frac{1}{3}$
C	300	$33\frac{1}{3}$
		Estate: 100

The *Kethuboth* rule

Case 2: A and B receive 125 together, A concedes 25 to B, B claims all, so the CG-rule gives 25 to B and divides what is left equally.

Claimants	Claims	*Kethuboth*
A	100	50
B	200	75
C	300	75
		Estate: 200

The *Kethuboth* rule

Case 2: A and B receive 125 together, A concedes 25 to B, B claims all, so the CG-rule gives 25 to B and divides what is left equally.

The same is true for A and C.

Claimants	Claims	*Kethuboth*
A	100	50
B	200	75
C	300	75
		Estate: 200

The *Kethuboth* rule

Case 2: A and B receive 125 together, A concedes 25 to B, B claims all, so the CG-rule gives 25 to B and divides what is left equally.

The same is true for A and C.

B and C receive 150 together, each claims all, so the CG-rule gives them equal shares.

Claimants	Claims	*Kethuboth*
A	100	50
B	200	75
C	300	75
		Estate: 200

The *Kethuboth* rule

Case 3: A and B receive 150 together, A concedes 50 to B, B claims all, so the CG-rule gives 50 to B and divides the 100 that is left equally.

Claimants	Claims	*Kethuboth*
A	100	50
B	200	100
C	300	150
		Estate: 300

The *Kethuboth* rule

Case 3: A and B receive 150 together, A concedes 50 to B, B claims all, so the CG-rule gives 50 to B and divides the 100 that is left equally.

A and C receive 200 together, A concedes 100 to C, C claims all, so the CG-rule gives 100 to C and divides the 100 that is left equally.

Claimants	Claims	*Kethuboth*
A	100	50
B	200	100
C	300	150
		Estate: 300

The *Kethuboth* rule

Case 3: A and B receive 150 together, A concedes 50 to B, B claims all, so the CG-rule gives 50 to B and divides the 100 that is left equally.

A and C receive 200 together, A concedes 100 to C, C claims all, so the CG-rule gives 100 to C and divides the 100 that is left equally.

B and C receive 250 together, B concedes 50 to C, C claims all, so the CG-rule gives 50 to C and divides the 200 that is left equally.

Claimants	Claims	*Kethuboth*
A	100	50
B	200	100
C	300	150
		Estate: 300

The proportional rule

Consider the problem with claims $(100, 200, 300)$ and all possible estates $0 \leq h \leq 600$.

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The **proportional rule** for an estate h is:

$$(s_A, s_B, s_C) = (100\lambda, 200\lambda, 300\lambda)$$

where λ is chosen so that $s_A + s_B + s_C = h$.

The *Kethuboth* rule: a formula

The ***Kethuboth* rule** for an estate h at most half the total claim is:

$$(s_A, s_B, s_C) = (\min\{\lambda, 50\}, \min\{\lambda, 100\}, \min\{\lambda, 150\})$$

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So, if $h = 100$ then $\lambda = 33\frac{1}{3}$; if $h = 200$ then $\lambda = 75$; and if $h = 300$ then $\lambda = 150$.

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The ***Kethuboth* rule** for an estate h at least half the total claim,

calculate the losses for $d_1 + d_2 + \cdots + d_n - h$,
subtract them from the claims.

So, if $h = 500$ then $(s_A, s_B, s_C) = (66\frac{2}{3}, 166\frac{2}{3}, 266\frac{2}{3})$; and if $h = 400$ then $(s_A, s_B, s_C) = (50, 125, 225)$.

The moral of this tale

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There are infinite numbers of coherent rules . . . but not all seemingly reasonable rules are coherent (as we will see)!

Contents

1 First Presidential Veto

2 Why Proportionality?

3 Fair Representation

4 Allocating Kidneys

Webster's method

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Total	9,496,036	49.000	49

But each state must have at least one representative:

State	Population (of 1900)	Quota	Rule
New York	7,264,183	37.522	37
Wyoming	92,531	0.478	1
Total	7,356,714	38.000	38

Webster's method

The rule, where $[x]$ means round x to the nearest integer:

$$(s_{NY}, s_{IA}) = \big(\max\{1, [p_{NY}\lambda]\}, \quad \max\{1, [p_{IA}\lambda]\}\big)$$

where λ is chosen so that $s_{NY} + s_{IA} = 49$.

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So in general, when h seats are to apportioned and the populations of n states are $(p_1, p_2, \ldots, p_n)$, the method is Daniel Webster's:

$$(s_1, s_2, \ldots, s_n) = \big(\max\{1, [p_1\lambda]\}, \max\{1, [p_2\lambda]\}, \ldots, \max\{1, [p_n\lambda]\}\big)$$

$$\text{where } \lambda \text{ is chosen so that } s_1 + s_2 + \cdots s_n = h.$$

Webster's method in his words, April 5, 1832

"To apportion is to distribute by right measure, to set off in just parts, to assign in due and proper proportion. . . [The] apportionment of representative power can never be precise and perfect. . . That which cannot be done perfectly must be done in a manner as near perfections as can be. . . Let the rule be that the population of each State be divided by a common divisor, and, in addition to the number of members resulting from such a division, a member shall be allowed to each State whose fraction exceeds a moiety of the divisor."

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	Quota	Hamilton
NY	37.606	38
PA	32.625	33
IA	11.554	11
VA	9.599	10
NE	5.520	5
ME	3.595	3
OR	2.141	2
VT	1.779	2
⋮	⋮	⋮
Sum	386	386

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Sum	386	386

	Quota	Hamilton
NY	37.432	37
ME	3.568	4
Sum	41	41

The "Alabama paradox"

House size	350-82	383-385	386	387-88	389-90	391-400
Maine's seats	3	4	3	4	3	4

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"Not only is Maine subjected to the assaults of the chairman of this committee [who had chosen $h = 386$], but it does seem as though mathematics and science had combined to make a shuttlecock and battledore of the state of Maine in connection with the scientific basis upon which this bill is presented . . . In Maine comes and out Maine goes . . .

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Coherent rules guarantee that when the size of the House h increases, no state can lose seats.

Coherent methods are "divisor methods"

All coherent apportionment methods must be one of these:

$$(s_1, s_2, \ldots, s_n) = \big(\max\{1, \langle p_1\lambda\rangle\},\ \max\{1, \langle p_2\lambda\rangle\}, \ldots, \max\{1, \langle p_n\lambda\rangle\}\big)$$

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$\langle x\rangle$ is defined by a **threshold** fixed in each interval $[0,1]$, $[1,2]$, $[2,3]$, $\ldots$, $[n, n+1], \ldots$: above it round-up, below it round-down, at the threshold do either.

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Hill's method, cleverly baptized the "method of equal proportions," is the law of the land in the United States since 1940: its thresholds are the geometric means $\sqrt{n(n+1)}$ of the end points of the intervals $[n, n+1]$.

The five traditional divisor methods

Five divisor methods were considered in the U.S. debate over which method should be used (1920's and 1930's):

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- Daniel Webster's: *round at the arithmetic mean*;
- Thomas Jefferson's: *round down* (used, rejected, accused of "*committing a classic rape on a cloud of statistics right in the face of the House*").

1900 apportionments

They give very different results! Going from left to right, bigger states more favored and smaller states less favored.

	Quota	Adams	Dean	Hill	Webstr	Jeffrsn	Hamiltn
NY	37.606	36	37	37	37	39	38
PA	32.625	31	32	32	32	34	33
IA	11.554	11	11	11	12	12	11
NE	5.520	6↗	5	6↗	5	5	5
ME	3.595	4	4	4	4↗	3	3
OR	2.141	3↗	2	2	2	2	2
VT	1.779	2	2	2	2↗	1	2
UT	1.425	2	2↗	1	1	1	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Total	386.000	386	386	386	386	386	386

Willcox *vs.* Huntington and the mathematicians

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> *The issue and the struggle underlying reapportionment is between the large States with large cities on one side and the rural and agricultural States on the other side. That thread of controversy runs through all the political struggles evidenced in this House. . . The issue grows more and more menacing.*

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Walter F. Willcox (1861-1964) of Cornell was president at different times of the American Economic, Statistical and Sociological Associations, and a great walker who remarked at the end of his life, "Unfortunately there is some danger that I will be remembered more for my feet than for my head." He championed Webster from 1900 to 1952. His main reason was summarized in 1915, in his presidential address to the AEA:

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> *The use of [Hill's method] has recently been advocated. To use it . . . would . . . result in defeating the main object of the Constitution, which is to hold the scales even between the small and the large states. For the use of [it] inevitably favors the small state.*

Willcox *vs.* Huntington and the mathematicians

Edward V. Huntington (1874-1952) of Harvard was at different times president of the Mathematical Association of America, vice-president of the American Mathematical Society and the American Association for the Advancement of Science, a charming and witty person, and an adept expert witness. He led the mathematicians in support of Hill's method:

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> *[S]tatistical experts . . . who have examined [Hill's method] have pronounced it the only scientific method. . . The method of [Webster] has a distinct bias in favor of the larger states, while the method of [Dean] has a similar bias in favor of the smaller states. Between these two methods stands the method of [Hill] which has been shown to have no bias in favor of the either the larger or the smaller states.*

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The two gave identical results in 1930: no choice was necessary. Hill gave one more seat to Democratic Arkansas, one less to Michigan in 1940. Hill was chosen.

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What would they have said had there been an even number of methods?

Which *is* unbiased?

The answer depends upon what bias means. Willcox had the right idea; the mathematicians neglected to define the concept.

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Theorem

Webster's is the unique unbiased divisor method.

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There have been 22 apportionments in U.S. history.

	Adams	Dean	Hill	Wbstr	Jffrsn
Number times small favored	22	17	15	10	0
Average % bias favor small	18.7	5.3	3.4	0.6	-16.2
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Walter Willcox's view was *true*: **Webster's** is the method to use!

Senator Arthur Vandenberg

April 19, 1929 address to the Senate:

> *"To identify any one method in this permanent act . . . would be to assume that science itself has traversed the subject with finality. Science is not thus static . . . The last word by no means has been spoken . . . A permanent ministerial apportionment act should be susceptible of accommodation to the progressive state of knowledge."*

Contents

1 First Presidential Veto

2 Why Proportionality?

3 Fair Representation

4 Allocating Kidneys

Organ transplants

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UNOS established a rule for each type of available kidney that assigns points to waiting patients.

UNOS priorities

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UNOS priorities determined by the total points of a patient: his "score" (all points that are independent of time of waiting) plus his "bonus."

The UNOS rule not coherent

Postulate four patients, A, B, C, and D, listed in order from the one who has waited the most to the one who has waited the least, and suppose their scores are as given.

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Incoherent, so inequitable: Yet, it is a simple matter to define a coherent rule!

HOW TO ELECT and RANK

Part 2: Representing, electing and ranking

Michel Balinski
École Polytéchnique and CNRS

Cornell University
September 26, 2007

REPRESENTING, ELECTING, and RANKING

Lecture 1: Why the current method of apportioning United States Representatives to the respective States is not equitable . . . *and what to do about it*.

Lecture 2: Why the "first-past-the-post" method of electing one among several candidates—the most used method of all—is seriously defective, often elects the "wrong" candidate . . . *and what to do about it*.

Lecture 3: Why blatant political gerrymandering is unavoidable in today's system . . . *and what to do about it*.

The sorry state of representation in the United States

- A minority of voters can (and has) elected a U.S. President.
- 50 Senators represent 16% of the population,
- 50 Senators represent 84% of the population.
- 51 Democratic Senators represent 58% of the population,
- 49 Republican Senators represent 42% of the population.
- In fact, as I will argue, a minority of voters can elect a majority of the U.S. House of Representatives (and probably has).

The sorry state of representation elsewhere

In the United Kingdom

The "winners" of the last six British elections:

	1983	1987	1992	1997	2001	2005
Votes	42.4%	42.2%	41.9%	43.2%	40.7%	35.2%
Seats	61.1%	57.8%	51.6%	63.4%	62.5%	55.1%

2005 election:

	Votes	Seats
Labour	35.2%	55.1%
Conservatives	32.3%	30.7%
Liberals	22.0%	9.6%

The sorry state of representation elsewhere

In France

Changes in the electoral law: 1982, 1985, 1991, 1998, 1999, 2000, 2003 . . .

Today's Assemblée Nationale districts drawn in 1986 on the basis of the census of 1982. By the last available data (based on 1999 census) populations of districts are:

2^{nd} Lozère	2^{nd} Val d'Oise	1^{st} Var	6^{th} Var
34,374	188,200	73,946	180,153

Contents

1 The Problem: How to Elect?

2 The Traditional View

3 The Majority Judgement

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Grover Cleveland	5,540,309	168

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Ralph Nader	2,882,955	0	97,488

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This violates Kenneth Arrow's famous *independence of irrelevant alternatives* condition.

French presidential elections

Constitution of the 5th Republic:

> *The president of the Republic is elected by the absolute majority of the votes. If it is not obtained in the first round of the election, a second round is held two Sundays later. The only two candidates who may present themselves, after the eventual withdrawal of more favored candidates, are those who have the largest number of votes in the first round.*

This is direct popular election: no Electoral College.

France 2002 presidential election

First round results (16 candidates, 72% participation):

Chirac	Le Pen	Jospin	Bayrou	Laguiller	Chévènement
19,88%	16,86%	16,18%	6,84%	5,72%	5,33%

Mamère	Besancenot	Saint-Josse	Madelin	Hue	Mégret
5,25%	4,25%	4,23%	3,91%	3,37%	2,34%

Taubira	Lepage	Boutin	Gluckstein
2,32%	1,88%	1,19%	0,47%

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Second round results (80% participation):

Chirac	Le Pen
82,21%	17,79%

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Chirac	Jospin
?	?

France 2002 election: minus Taubira (erstwhile socialist), plus Pasqua (old Chirac ally)

First round results:

Chirac	Le Pen	Jospin	Bayrou	Laguiller	Chévènement
16,38%	16,86%	17,18%	6,84%	5,72%	5,33%

Mamère	Besancenot	Saint-Josse	Madelin	Hue	Mégret
5,25%	4,25%	4,23%	3,91%	3,37%	2,34%

Pasqua	Lepage	Boutin	Gluckstein
3,70%	1,88%	1,19%	0,47%

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Second round results:

Jospin	Le Pen
>75%	<25%

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The French voters are very aware that the votes for Chirac against Le Pen in 2002 had very different "values": the left voted heavily for him . . . holding their noses.

France 2007 presidential election

First round results (12 candidates, 84% participation):

Sarkozy	Royal	Bayrou	Le Pen	Besancenot	de Villiers
31,18%	25,87%	18,57%	10,44%	4,08%	2,23%

Buffet	Voynet	Laguiller	Bové	Nihous	Schivardi
1,93%	1,57%	1,33%	1,32%	1,15%	0,34%

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Second round results (84% participation):

Nicolas Sarkozy	18,983,138	53.06%
Ségolène Royal	16,790,440	46.94%

France 2007 presidential election

First round results (12 candidates, 84% participation):

Sarkozy	Royal	Bayrou	Le Pen	Besancenot	de Villiers
31,18%	25,87%	18,57%	10,44%	4,08%	2,23%

Buffet	Voynet	Laguiller	Bové	Nihous	Schivardi
1,93%	1,57%	1,33%	1,32%	1,15%	0,34%

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The lessons of 2002: higher participation, more "strategic voting."

2007 election, poll 3 weeks before first round

	Bayrou	Le Pen	Royal	Sarkozy
Bayrou	—	84%	57%	54%
Le Pen	16%	—	25%	16%
Royal	46%	75%	—	46%
Sarkozy	46%	84%	54%	—

2007 election, poll 3 weeks before first round

	Bayrou	Le Pen	Royal	Sarkozy
Bayrou	—	84%	57%	54%
Le Pen	16%	—	25%	16%
Royal	46%	75%	—	46%
Sarkozy	46%	84%	54%	—

The poll asserted

- 72% of those for Royal would vote for Bayrou against Sarkozy,
- 75% of those for Sarkozy would vote for Bayrou against Royal.

2007 election, poll 3 weeks before first round

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The French electoral system—and any "first-past-the-post" system—can fail to elect the *Condorcet-winner*, the candidate who defeats all others in head-to-head confrontations.

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Voter's lists and the *Condorcet paradox (1786)*

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Observe, first, there may be no Condorcet-winner:

29%	32%	38%
A	B	C
B	C	A
C	A	B

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$$A(67\%) \succ B(61\%) \succ C(70\%) \succ A$$

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This is the famous *Condorcet paradox*.

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- give to every candidate a point for each candidate he defeats in a head-to-head race (a point to both if they are tied),
- elect the candidate with the most points.

This mechanism is known today as *Copeland's method* (1951), but it does not designate an unequivocal winner either.

Condorcet-winners are not necessarily legitimate

Suppose $3n$ voters have the lists

n	n	n
A	B	C
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alternatively, the opinions of these voters cancel each other out.

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Suppose the electorate's lists are as follows:

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But, 39% on the left and 12% on the right cancel each other:

21%	0%	0%	0%	0%	28%
A	B	C	A	C	B
B	C	A	C	B	A
C	A	B	B	A	C

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The obvious winner is B.

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- The *Borda-ranking* ranks the candidates according to their Borda-counts.

It clearly cancels properly!

Borda-winners are not necessarily legitimate

Suppose the electorate's lists are as follows:

33%	3%	16%	18%	30%
A	B	A	C	B
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The Borda-winner is A since the Borda-ranking is

$$A(128) \succ B(117) \succ C(55)$$

e.g., A's Borda-count $= 2 \times (33 + 16) + 30$.

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But if C withdraws the lists become:

49%	51%
A	B
B	A

so B is the Borda-winner!

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Theorem (Arrow's impossibility)

No method of election meets these requirements.

This has usually been interpreted as meaning: there can be no satisfactory mechanism for amalgamating the wants, wishes or opinions of an electorate.

The modern debate

Nevertheless, in today's ongoing debates about voting

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Why? Presumably, its manipulability.

Manipulability

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Laplace had imagined that voters give number grades to candidates, then justified Borda's method with a probabilistic argument, but he ultimately rejected it:

> *[Borda's method] would be without a doubt the best if considerations alien to the merit [of a candidate] did not influence the choice of the electors, even the most honest ones, and did not determine them to rank last the most dangerous opponents to their favorites . . .*

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Theorem (Gibbard-Satterthwaite's impossibility)

There is no method for which a voter's best strategy is to vote his convictions.

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A different view and new theory eliminates the impossibilities

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Bulletin de vote : Élection du Président de la République 2007

Pour la France,
ayant pris tous les éléments en compte,
je juge en conscience que ce candidat serait :

	Très Bien	Bien	Assez Bien	Passable	Insuffisant	A Rejeter
Olivier Besancenot						
Marie-George Buffet						
Gérard Schivardi						
François Bayrou						
José Bové						
Dominique Voynet						
Philippe de Villiers						
Ségolène Royal						
Frédéric Nihous						
Jean-Marie Le Pen						
Arlette Laguiller						
Nicolas Sarkozy						

Cochez une seule mention dans la ligne de chaque candidat.
Ne pas cocher une mention dans la ligne d'un candidat revient à le Rejeter.

Ramon Llull's 1299 charge to the voter

"[It] is necessary to ascertain that in the election three things should be considered, of which the first is honesty and holiness of life, the second is knowledge and wisdom, and the third is a suitable disposition of the heart. Each person having a vote in the chapter should take an oath by the holy gospels of God to consider these three things and to always elect the person in whom they are best [embodied]."

A different view and new theory erases the impossibilities

Ballot: Election of President of France 2007

To be president of France,
having taken into account all considerations,
I judge, in conscience, that this candidate would be:

	Excellent	Very Good	Good	Acceptable	Poor	To Reject
Olivier Besancenot						
Marie-George Buffet						
Gérard Schivardi						
François Bayrou						
José Bové						
Dominique Voynet						
Philippe de Villiers						
Ségolène Royal						
Frédéric Nihous						
Jean-Marie Le Pen						
Arlette Laguiller						
Nicolas Sarkozy						

Check one single annotation in the line of each candidate.

No annotation checked in the line of a candidate means to Reject the candidate.

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- When two candidates have the same majority-grade, one must be ranked ahead of the other *unless* they have identical sets of grades.

Consequence: This method—the *Majority Judgement*—satisfies Arrow's independence of irrelevant alternatives condition.

The charge to voters, and the common language

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The words used in the French ballot do constitute—for France—a common language. Their translations do not necessarily yield an adequate common language.

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Excellent	Very Good	Good	Acceptable	Poor	To Reject
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The majority-grade: combats strategic voting

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Suppose a voter judged this candidate "Very Good": he/she has no reason to exaggerate the grade by giving an "Excellent."

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Suppose a voter judged this candidate "Acceptable": he/she has no reason to exaggerate the grade by giving a "Poor" or "to Reject."

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The majority-grade uniquely satisfies these and other properties.

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If a candidate's % *better than MG* is greater than his/her % *worse than MG* then it is a *MG+*

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$A \succ_{maj} B$ when

- A's MG higher than B's,
- A's is $MG+$, B's is $MG-$,
- A's and B's is $MG+$, A's % *better than MG* greater than B's
- A's and B's is $MG-$, B's % *worse than MG* greater than A's.

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- Carried out under identical conditions: ballots filled out in booths, inserted in envelopes and deposited in transparent urns.

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- 2,360 voted officially, 1,752 (74%) participated in experiment.
- Television interviews prove the satisfaction of voters.

Results French Presidential elections, Orsay 3 Bureaux

	Excel.	V.Good	Good	Accpt.	Poor	Rej.	Blank
	13.6%	30.7%	25.1%	14.8	8.4%	4.5%	2.9%
	16.7%	22.7%	19.1%	16.8%	12.2%	10.8%	1.8%
	19.1%	19.8%	14.3%	11.5%	7.1%	26.5%	1.7%
Voynet	2.9%	9.3%	17.5%	23.7%	26.1%	16.2%	4.3%
Besancenot	4.1%	9.9%	16.3%	16.0%	22.6%	27.9%	3.2%
Buffet	2.5%	7.6%	12.5%	20.6%	26.4%	26.1%	4.3%
Bové	1.5%	6.0%	11.4%	16.0%	25.7%	35.3%	4.2%
Laguiller	2.1%	5.3%	10.2%	16.6%	25.9%	34.8%	5.3%
Nihous	0.3%	1.8%	5.3%	11.0%	26.7%	47.8%	7.2%
Villiers	2.4%	6.4%	8.7%	11.3%	15.8%	51.2%	4.3%
Schivardi	0.5%	1.0%	3.9%	9.5%	24.9%	54.6%	5.8%
Le Pen	3.0%	4.6%	6.2%	6.5%	5.4%	71.7%	2.7%

Results French Presidential elections, Orsay 3 Bureaux

	Excel.	V.Good	Good	Accpt.	Poor	Rej.	Blank
Bayrou	13.6%	30.7%	25.1%	14.8	8.4%	4.5%	2.9%
Royal	16.7%	22.7%	19.1%	16.8%	12.2%	10.8%	1.8%
Sarkozy	19.1%	19.8%	14.3%	11.5%	7.1%	26.5%	1.7%
Voynet	2.9%	9.3%	17.5%	23.7%	26.1%	16.2%	4.3%
Besancenot	4.1%	9.9%	16.3%	16.0%	22.6%	27.9%	3.2%
Buffet	2.5%	7.6%	12.5%	20.6%	26.4%	26.1%	4.3%
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Laguiller	2.1%	5.3%	10.2%	16.6%	25.9%	34.8%	5.3%
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Schivardi	0.5%	1.0%	3.9%	9.5%	24.9%	54.6%	5.8%
Le Pen	3.0%	4.6%	6.2%	6.5%	5.4%	71.7%	2.7%

Majority-grades and majority-ranking, French Presidential Elections, Orsay 3 Bureaux

		Higher M-G	The M-G	Lower M-G	Official vote	Ntnl vote
3	Bayrou	44.3%	Good+	30.6%	25.5%	18.6%
2	Royal	39.4%	Good-	41.5%	29.9%	25.9%
1	Sarkozy	38.9%	Good-	46.9%	29.0%	31.2%
8	Voynet	29.8%	Acceptable-	46.6%	1.7%	1.6%
5	Besancenot	46.3%	Poor+	31.2%	2.5%	4.1%
7	Buffet	43.2%	Poor+	30.5%	1.4%	1.9%
10	Bové	34.9%	Poor-	39.4%	0.9%	1.3%
9	Laguiller	34.2%	Poor-	40.0%	0.8%	1.3%
11	Nihous	45.0%	To reject	-	0.3%	1.2%
6	Villiers	44.5%	To reject	-	1.9%	2.2%
12	Schivardi	39.7%	To reject	-	0.2%	0.3%
4	Le Pen	25.7%	To reject	-	5.9%	10.4%

Could strategic voting have made Royal the winner?

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Could strategic voting have made Royal the winner?

Yes, if *many*! If those voters who graded Royal above Bayrou did as postulated below ... they would have failed. A voter who can raise/lower one candidate's grade cannot lower/raise the other's. Polls estimated at 20% the strategic $\neq$ conviction votes.

	Excell-ent	Very good	Good	Accept-able	Poor	To reject	Vote strategy
2.8%				R	B	B	0%
6.3%	R	B$\rightarrow$	$\rightarrow$	$\rightarrow$ |			33%
6.9%		R	B$\rightarrow$	$\rightarrow$ |			33%
2.4%		| $\leftarrow$	$\leftarrow$R	B			33%
3.2%	R		B$\rightarrow$	$\rightarrow$ |			67%
2.1%		| $\leftarrow$	$\leftarrow$R		B	B	67%
9.2%	R	R		B	B	B	cannot

Wines

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Œnologists argue that the sum misses the point because it "has difficulty in detecting exceptional wines by overly favoring those that are 'taste-wise correct.' "

Moreover, expert judges—I have been assured—taste, decide on the final score, . . . then attribute points to characteristics that add to the score they wished.

The Union Internationale des Œnologues method

		Excllt	V.gd	Gd	Pss	Inad	Mdcr	Bad
Aspect	Limpidity	6	[5]	4	3	2	1	0
	Nuance	[6]	5	4	3	2	1	0
	Intensity	6	5	[4]	3	2	1	0
Aroma	Frankness	[6]	5	4	3	2	1	0
	Intensity	8	[7]	6	5	4	2	0
	Finesse	8	7	[6]	5	4	2	0
	Harmony	[8]	7	6	5	4	2	0
Taste	Frankness	6	[5]	4	3	2	1	0
Flavor	Intensity	8	[7]	6	5	4	2	0
	Body	8	7	[6]	5	4	2	0
	Harmony	[8]	7	6	5	4	2	0
	Persistence	8	[7]	6	5	4	2	0
	After-taste	8	[7]	6	5	4	2	0
	Glbl opinion	[8]	7	6	5	4	2	0

Score: 90

The majority judgement: wines

For you, this wine is:

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For you, this wine is:

Excellent, *Very good*, *Good*, *Passable*, *Inadequate*, *Mediocre*, or *Bad*.

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Typically, there are five judges. For any wine, the grades may be listed from best to worst.

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For you, this wine is:

Excellent, *Very good*, *Good*, *Passable*, *Inadequate*, *Mediocre*, or *Bad*.

Typically, there are five judges. For any wine, the grades may be listed from best to worst.

Bordeaux
Very good
Good
Good
Good
Bad

The Bordeaux's majority-grade is *Good*.

The majority judgement: wines

However, sometimes there may an even number of judges: there is no median or middle grade. The lower-middle grade must be given.

The majority judgement: wines

However, sometimes there may an even number of judges: there is no median or middle grade. The lower-middle grade must be given.

Arbois
Good
Good
Good
Passable
Passable
Passable

The Arbois's majority-grade is *Passable*.

The majority judgement: wines

However, sometimes there may an even number of judges: there is no median or middle grade. The lower-middle grade must be given.

Arbois
Good
Good
Good
Passable
Passable
Passable

The Arbois's majority-grade is *Passable*. Why?

The majority judgement: wines

Arbois	Beaune
Good	*Excellent*
Good	*Very good*
Good	*Very good*
Passable	*Inadequate*
Passable	*Mediocre*
Passable	*Bad*

The majority judgement: wines

Arbois	Beaune
Good	*Excellent*
Good	*Very good*
Good	*Very good*
Passable	*Inadequate*
Passable	*Mediocre*
Passable	*Bad*

The Arbois should rank ahead of the Beaune, Arbois $\succ$ Beaune, because all of the Arbois's grades belong to the middle interval of the Beaune's grades:

$$\cdots \succ \textit{Very good} \succ \textit{Good} \succ \textit{Passable} \succ \textit{Inadequate} \succ \cdots$$

The majority judgement: ranking three wines

Anjou	Bourgogne	Chablis
Very good	*Excellent*	*Excellent*
Very good	*Very good*	*Excellent*
Good	*Good*	*Good*
Good	*Good*	*Passable*
Passable	*Mediocre*	*Mediocre*

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Anjou	Bourgogne	Chablis
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The majority judgement: ranking three wines

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Very good	*Excellent*
Very good	*Very good*
Passable	*Mediocre*

Anjou	Bourgogne
Very good	*Excellent*
Passable	*Mediocre*

Therefore: Anjou $\succ$ Bourgogne $\succ$ Chablis

The majority judgement: ranking and tie-breaking

The majority-ranking deduced from principles here in the context of wines, implies the tie-breaking rule among candidates having the same grade described earlier for voting (where there are so many judges that it is reasonable to assume that the rule will encounter no ties among the percentages of the grades of the candidates).

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In fact, the majority-ranking distinguishes between any two competitors unless they have an identical set of grades.

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HOW TO ELIMINATE GERRYMANDERING

Part 3: Representing, electing and ranking

Michel Balinski
École Polytéchnique and CNRS

Cornell University
September 27, 2007

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Lecture 3: Why blatant political gerrymandering is unavoidable in today's system . . . *and what to do about it*.

The original gerrymander



Gerrymandering

Political gerrymandering:

> *[The] practice of dividing a geographical area into electoral districts, often of highly irregular shape, to give one political party an unfair advantage by diluting the opposition's voting strength.*

–from *Black's Law Dictionary* as quoted by Justice Antonin Scalia.

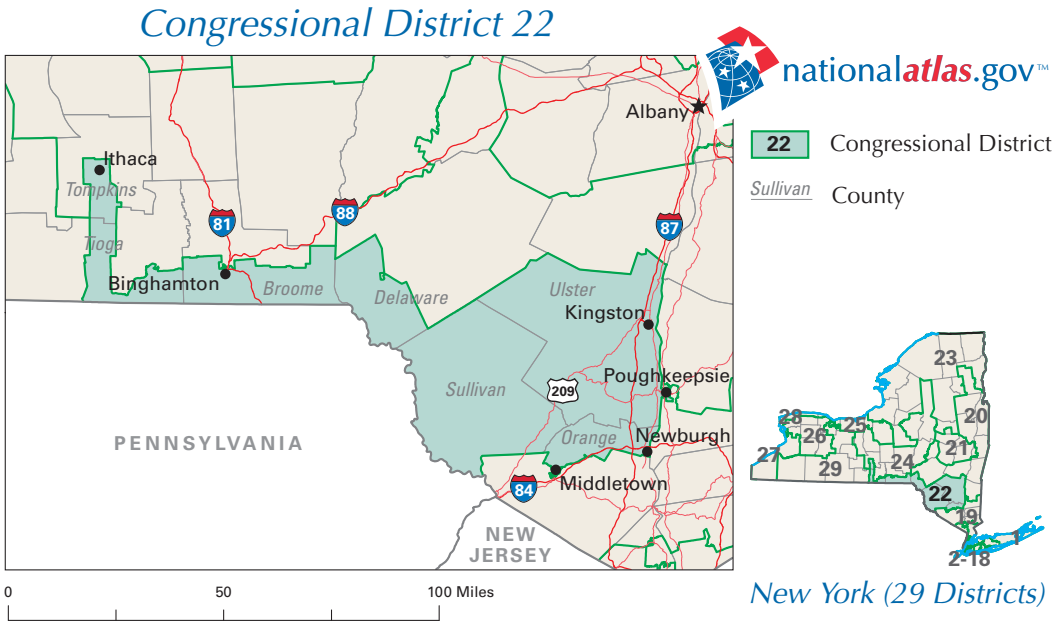
Contents

1 Electoral Realities

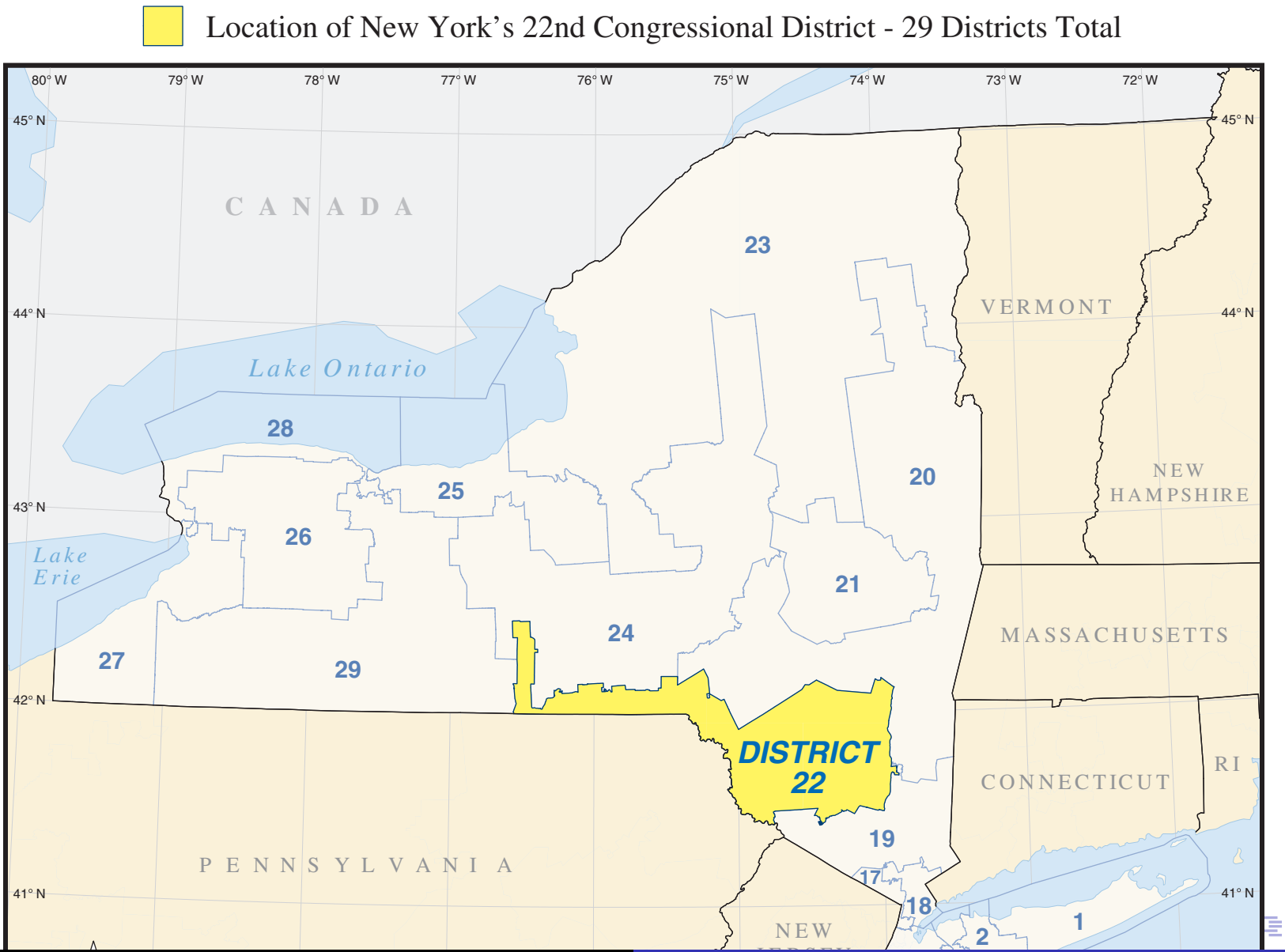
2 Fair majority voting

3 Biproportionality

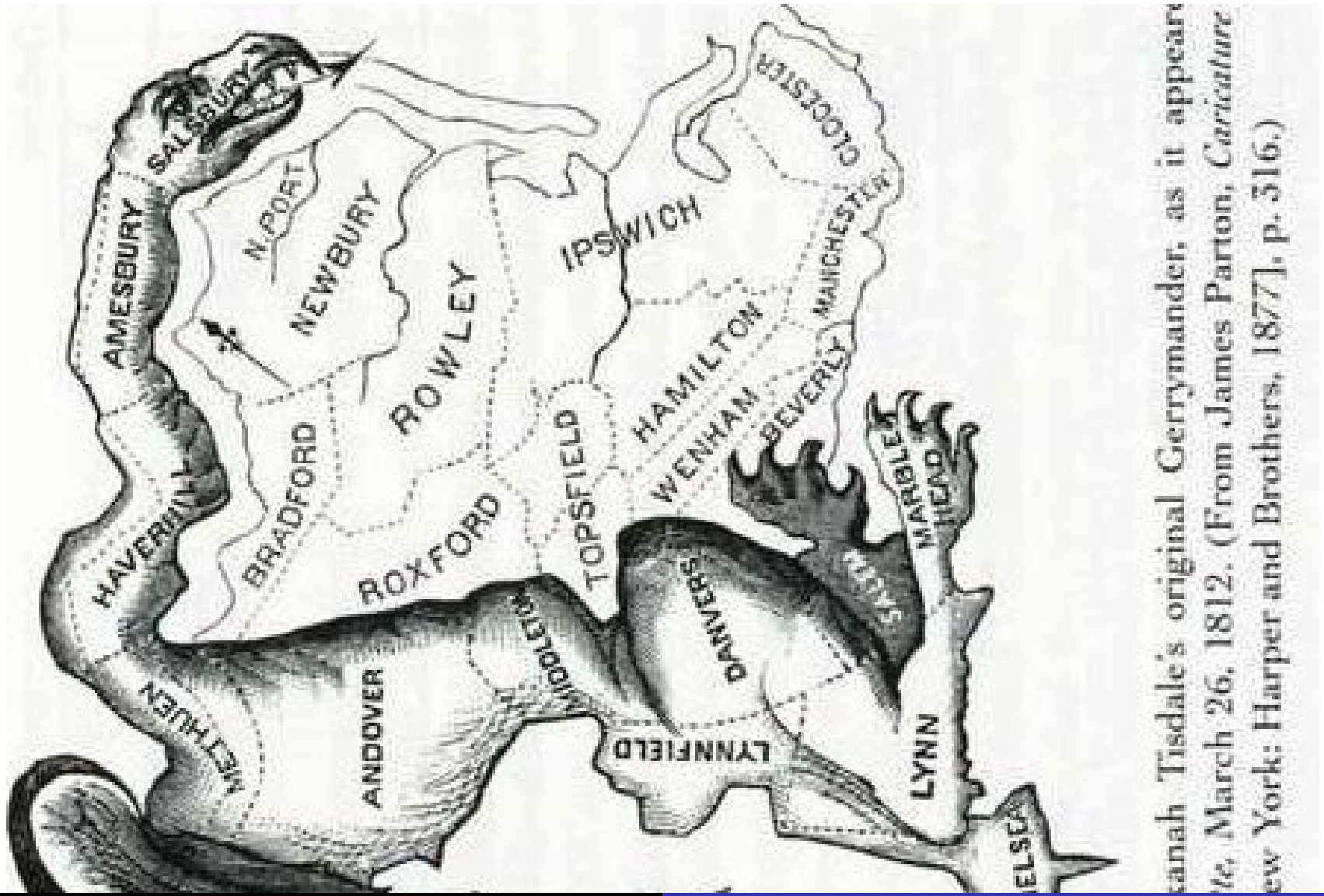
Ithaca's gerrymander



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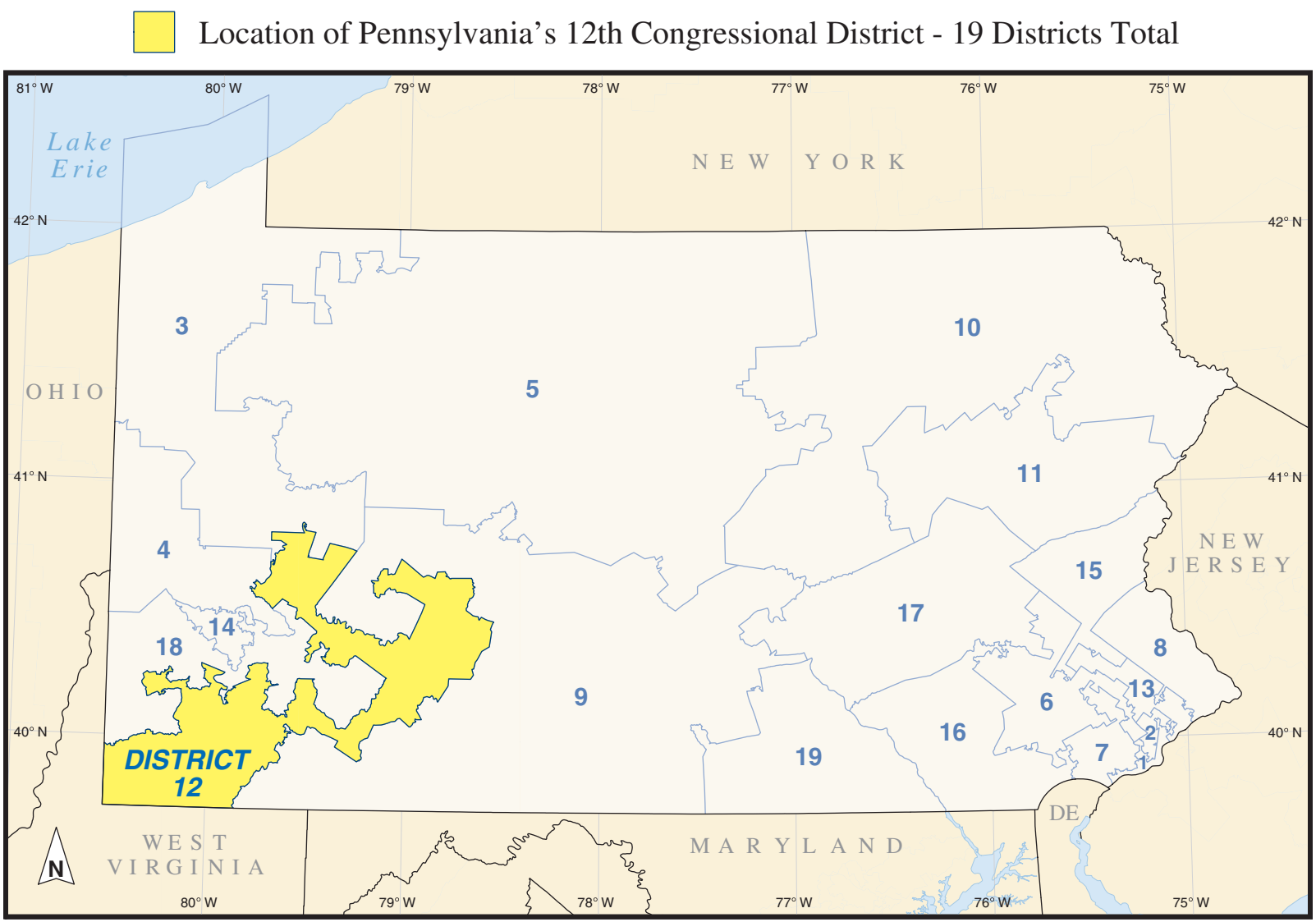


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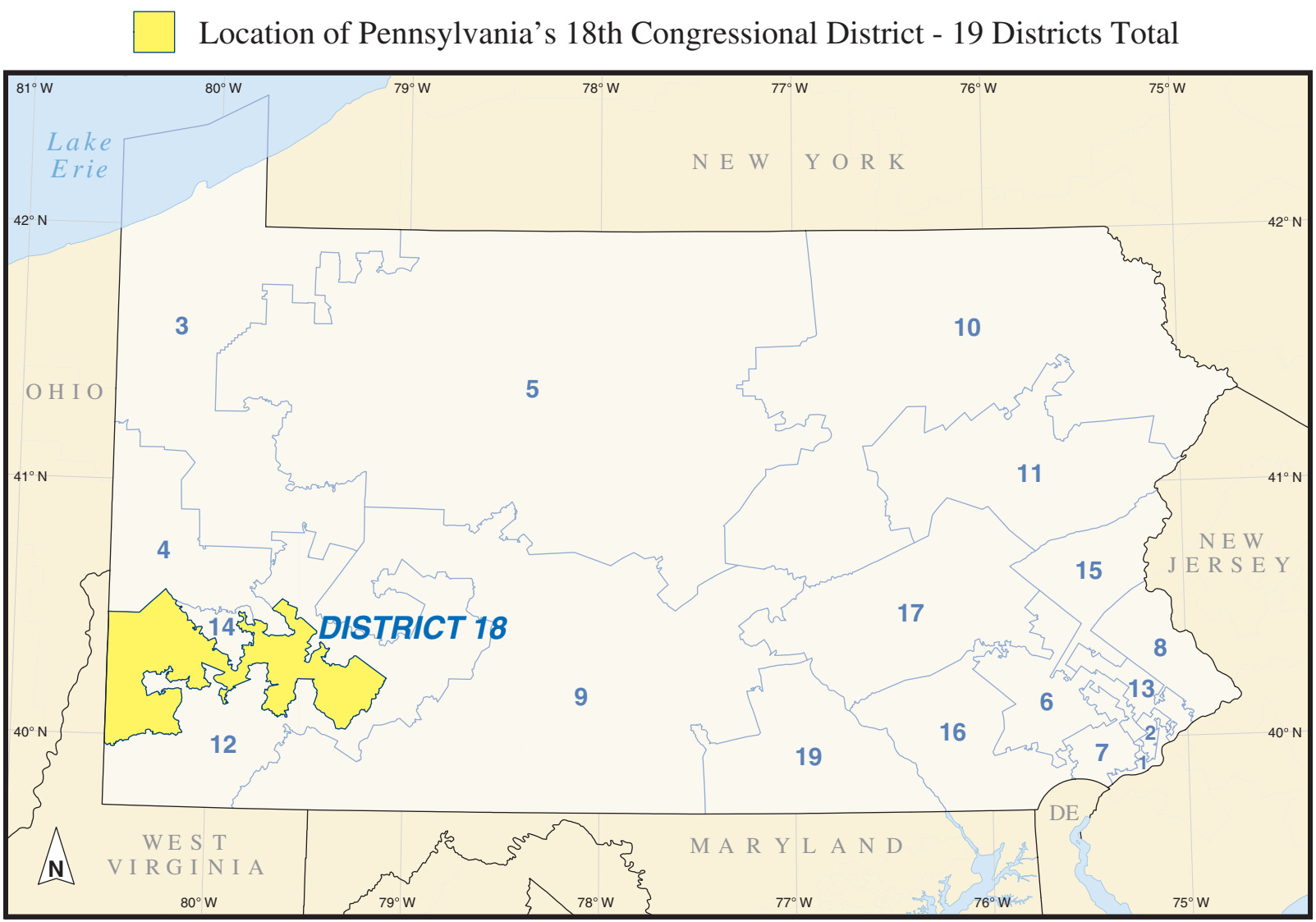


anah Tisdale's original Gerrymander, as it appear
e, March 26, 1812. (From James Parton, *Caricature*
ew York: Harper and Brothers, 1877, p. 316.)

Pennsylvania's gerrymander: upside-down Chinese dragon?



Pennsylvania's gerrymander: supine seahorse?



The rotten electoral state of the United States

	2002	2004	2006
Incumbent candidates	386	392	394
Incumbent candidates reelected	380	389	371
Incumbent candidates who lost to outsiders	4	3	23
Elected candidates ahead by $\geq$20% of votes	356	361	318
Elected candidates ahead by $\geq$16% of votes	375	384	348
Elected candidates ahead by $\leq$10% of votes	36	22	56
Elected candidates ahead by $\leq$6% of votes	24	10	39
Candidates elected without opposition	81	66	59
Republicans elected	228	232	202
Democrats elected	207	203	233

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Entirely possible for a majority party in the House to be elected by a minority!

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2002 Congressional elections:

- Michigan: Democratic vote exceeded Republican by 35,000, but only 6 Democrats elected to Republican's 9 Representatives.
- Maryland: Average vote of Republican winner 376,455, of Democratic winner 150,708.

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- Maryland: Average vote of Republican winner 376,455, of Democratic winner 150,708.

Connecticut:

- 2004: Democratic votes exceeded Republican by 156,000, yet only elected 2 to the Republican's 3 Representatives,
- 2006: 44% of votes gave Republicans only 1 seat (20%) to Democrat's 4.

The rotten electoral state of the United States

Massachusetts 2002, 2004, 2006:

- All 10 Representatives Democrats,
- respectively, 6, 5 and 7 elected without opposition.

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The new science of political gerrymandering

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And yet, every one of Texas's 32 districts had a census population of **651,619** or **651,620**: a perfectly carved state!

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Pennsylvania's governor was Republican, the party controlled state House and Senate.

The new science of political gerrymandering

The Pennsylvania redistricting story explains:

> *"Democrats rewrote the book when they did Georgia, and we would be stupid not to reciprocate . . . [the Pennsylvania redistricting] will make Georgia look like a picnic,"*

said the Chairman of the National Republican Congressional Committee.

Pennsylvania's governor was Republican, the party controlled state House and Senate.

Note in passing: Political gerrymandering is perfectly ecumenical.

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*Caliper Corp.'s *Maptitude for Redestricting* does this for $6,000.

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In 2002, 7 Democrats (1 unopposed) and 12 Republicans (4 unopposed) were elected.

Recourse in the Courts

The Democrats filed suit, claiming:

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A few more clicks of the mouse: each district's population either 646, 371 or 646,372. But

- supine seahorses and upside down Chinese dragons,
- 21 counties and 81 municipalities fractured.

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Justice Antonin Scalia announced the judgement (joined by only 3 other justices), concluding:

> *Eighteen years of essentially pointless litigation have persuaded us that Bandemer [1986] is incapable of principled application. We would therefore overrule that case, and decline to adjudicate these political gerrymandering claims. The judgement of the District Court is affirmed.*

One criterion accepted by the Court

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Only one criterion is accepted as capable of application, the numbers. *Kirkpatrick v. Preisler 394 U.S. 526 (1969)*:

> *[The] "nearly as practicable" standard requires that the State make a good-faith effort to achieve precise mathematical equality. Unless population variances among congressional districts are shown to have resulted despite such effort, the State must justify each variance, no matter how small.*

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The Supreme Court's decisions and dissenting opinions, taken together, have left a legacy of utter confusion.

Justice Harlan's charge

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Justice John Harlan was unusually prescient in a 1969 dissenting opinion:

> *[The] rule of absolute equality is perfectly compatible with "gerrymandering" of the worst sort. A computer may grind out district lines which can totally frustrate the popular will . . . The legislature must do more than satisfy one man, one vote; it must create a structure which will in fact as well as theory be responsive to the sentiments of the community. . . Even more than in the past, district lines are likely to be drawn to maximize the political advantage of the party temporarily dominant in public affairs.*

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From this perspective, many electors are very badly represented!

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Two rules decide which candidates are elected:

- The *requisite number* of representatives elected by each party is determined by Jefferson's method on the basis of total party votes.
- The candidates elected—exactly one in each district and the requisite number of each party—are determined by the procedure that is about to be described.

The problem

The 2004 Connecticut congressional elections: votes.

District	1st	2nd	3d	4th	5th	Total
Republican	73,273	165,558	68,810	149,891	165,440	622,972
Democratic	197,964	139,987	199,652	136,481	105,505	779,589

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The district-winners of the justified-votes are 3 Democrats and 2 Republicans: FMV elects them.

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The exact same solution: **it always is**.

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- Every candidate incited to seek as many votes as possible (as vs. "proportional representation").

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This is unavoidable. The evidence shows electorates are prepared to accept it.

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Mr. Schmidt was the resident of a district with 3 representatives; he regularly cast his votes for a party that never received enough votes in his district to elect one of its candidates. The Court ruled he was right!

Zürich city election of February 12, 2006

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Party→ Dist.↓	A	B	C	D	E	F	G	H	Seats
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4^{th}	2409	968	1092	842	440	342	230	111	10
5^{th}	3632	1642	3015	1499	837	618	323	144	17
6^{th}	2628	1972	754	572	708	615	154	333	16
7^{th}	2938	1630	1272	807	696	391	212	124	12
8^{th}	2976	2113	1039	661	777	631	191	328	19
9^{th}	1322	1025	307	219	494		43	208	10
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5^{th}	3632	1642	3015	1499	837	618	323	144	17
6^{th}	2628	1972	754	572	708	615	154	333	16
7^{th}	2938	1630	1272	807	696	391	212	124	12
8^{th}	2976	2113	1039	661	777	631	191	328	19
9^{th}	1322	1025	307	219	494		43	208	10
Seats	44	24	19	14	10	6	5	3	125

Biproportional apportionment

Biproportional apportionment

Multipliers can always be found to rescale rows (or votes in districts) and/or columns (or votes for parties) so that rounding the results to the nearest integers yields an apportionment that gives to each district and each party the seats it deserves.

The rescaling:

Biproportional apportionment

Multipliers can always be found to rescale rows (or votes in districts) and/or columns (or votes for parties) so that rounding the results to the nearest integers yields an apportionment that gives to each district and each party the seats it deserves.

The rescaling:

	A	B	C	D	E	F	G	H
1^{st}	3.92	2.12	3.00	1.72	1.02	0.45	0.42	0.23
2^{nd}	6.52	3.19	1.49	2.51	1.25	0.46	1.34	0.46
3^{rd}	5.08	1.57	0.86	1.96	0.79	0.22	2.18	0.27
4^{th}	3.61	1.47	1.64	1.28	0.67	0.59	0.44	0.17
5^{th}	5.45	2.49	4.52	2.27	1.27	1.06	0.61	0.22
6^{th}	5.51	4.17	1.58	1.21	1.49	1.48	0.41	0.70
7^{th}	4.48	2.51	1.94	1.24	1.07	0.68	0.41	0.19
8^{th}	6.27	4.49	2.19	1.41	1.65	1.53	0.51	0.70
9^{th}	3.27	2.56	0.76	0.55	1.23		0.13	0.52

Biproportional apportionment

The solution:

	A	B	C	D	E	F	G	H	
1^{st}	4	2	3	2	1	0	0	0	12
2^{nd}	7	3	1	3	1	0	1	0	16
3^{rd}	5	2	1	2	1	0	2	0	13
4^{th}	4	1	2	1	1	1	0	0	10
5^{th}	5	2	5	2	1	1	1	0	17
6^{th}	6	4	2	1	1	1	0	1	16
7^{th}	4	3	2	1	1	1	0	0	12
8^{th}	6	4	2	1	2	2	1	1	19
9^{th}	3	3	1	1	1		0	1	10
	44	24	19	14	10	6	5	3	125

Biproportional apportionment

Biproportional apportionment

	A	B	C	D	E	F	G	H	Seats
5^{th}	3632	**1642**	3015	1499	837	618	323	144	
	5	**2**	5	2	1	1	1	0	17
8^{th}	2976	2113	1039	661	777	**631**	191	328	
	6	4	2	**1**	2	**2**	1	1	19
9^{th}	1322	**1025**	307	219	494		43	208	
	3	**3**	1	1	1		0	1	10

Biproportional apportionment

	A	B	C	D	E	F	G	H	Seats
5^{th}	3632	**1642**	3015	1499	837	618	323	144	
	5	**2**	5	2	1	1	1	0	17
8^{th}	2976	2113	1039	661	777	**631**	191	328	
	6	4	2	**1**	2	**2**	1	1	19
9^{th}	1322	**1025**	307	219	494		43	208	
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Theorem

Such multipliers can always be found (for any number of parties and districts with any number of seats).

Biproportional apportionment

	A	B	C	D	E	F	G	H	Seats
5^{th}	3632	**1642**	3015	1499	837	618	323	144	
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Theorem

Such multipliers can always be found (for any number of parties and districts with any number of seats). They always yield the identical set of elected candidates.

Biproportional apportionment

	A	B	C	D	E	F	G	H	Seats
5^{th}	3632	**1642**	3015	1499	837	618	323	144	
	5	**2**	5	2	1	1	1	0	17
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9^{th}	1322	**1025**	307	219	494		43	208	
	3	**3**	1	1	1		0	1	10

Theorem

Such multipliers can always be found (for any number of parties and districts with any number of seats). They always yield the identical set of elected candidates. No other set of feasible candidates is coherent with the simple rounding rule ***for every pair of party-district lists****.*

Messenger's charge and Tocqueville's remark

Messenger's charge and Tocqueville's remark

The terms of Dr. Hiram Messenger's original gift to establish this series of lectures stated:

> *"... to provide a course of lectures ... for the special purpose of raising the moral standard of our political, business, and social life ..."*

Messenger's charge and Tocqueville's remark

The terms of Dr. Hiram Messenger's original gift to establish this series of lectures stated:

> *"... to provide a course of lectures ... for the special purpose of raising the moral standard of our political, business, and social life ..."*

Alexis de Tocqueville's remark in a January 5, 1851 letter to his cousin and close friend Gustave de Beaumont, puts this charge in perspective:

> *"How sad it is that everywhere on earth governments are always precisely as roguish as the morals of their subjects permit them to be!*
>
> *Their vices have found but that one limit."*

Some Biographical Information

Michel Balinski, École Polytechnique, Paris

Some biographical information about Michel Balinski is linked below:

- Brief Biography
- Curriculum Vitae
- List of Publications

Michel L. Balinski

Michel Louis Balinski (born 6 October 1933) is an applied mathematician, economist, operations research analyst and political scientist. American, educated in the United States, he has lived and worked primarily in the United States and France. He is known for his work in optimization (combinatorial, linear, nonlinear), convex polyhedra, stable matching, and the theory and practice of electoral systems, jury decision, and social choice. He is Directeur de recherche de classe exceptionnelle (emeritus) of the C.N.R.S. at the Ecole Polytechnique (Paris). He was awarded the John von Neumann Theory Prize by INFORMS in 2013.

Early Life

Michel Balinski was born in Geneva, Switzerland, the son of a Polish diplomat at the League of Nations and the grandson of the Polish bacteriologist and UNICEF founder Ludwik Rajchman. He was living with his grandparents in France when in 1940 the family fled the Nazis via Spain and Portugal to the United States. He graduated from the Edgewood School in Greenwich CT in 1950, earned a B.A. degree cum laude in mathematics at Williams College in 1954 and a M.Sc. in economics at the Massachusetts Institute of Technology in 1956. He completed a Ph.D. in mathematics under the supervision of Albert W. Tucker at Princeton University in 1959.

Career

After completing his Ph.D. Balinski remained at Princeton as a Research Associate then Lecturer in mathematics. From 1963 to 1965 he was Associate Professor of Economics at the Wharton School of the University of Pennsylvania. He was then appointed to the Graduate School of the City University of New York, first as Associate Professor then (as of 1969) Professor of Mathematics. One of his doctoral students at the City University was another noted mathematician, Louis Billera, through whom he has many academic descendants. In 1978 he was appointed Professor of Organization and Management and of Administrative Sciences at Yale.

In parallel with his academic work Balinski engaged in consulting as of the time he was a graduate student at Princeton. A participant in the beginnings of what became the consulting firm Mathematica, he was a Senior Consultant with the firm from 1962 to 1974. He also acted as a consultant elsewhere, including the Rand Corporation, Mobil Oil Research, the ORTF (Office de Radiodiffusion-Télévision Française), the Mayor's office of the City of New York (as member of the Operations Research Council), and Econ, Inc. From 1975 to 1977 he was Chairman of System and Decision Sciences at IIASA (International Institute for Applied Systems Analyses) at Laxenburg, Austria.

In 1980 Michel Balinski settled in France, becoming Directeur de Recherche de classe exceptionnelle of the CNRS (Centre National de Recherche Scientifique) at the Laboratoire d'Econométrie of the Ecole Polytechnique in 1983. Concurrently he was Leading Professor of Applied Mathematics and Statistics and of Economics at SUNY (State University of New York) Stony Brook (1983-1990), where he founded and was the first Director of the Institute for Decision Sciences (that has since become the Center for Game Theory in Economics).

Upon becoming Director of the Laboratoire d'Econométrie (1989 to 1999) he co-founded and co-directed the joint Ecole Polytechnique/Université de Paris 1 Ph.D. program "Modélisation et methodes mathématiques en économie: optimisation et analyse stratégiques," and its successor the joint Ecole Polytechnique/Université Pierre et Marie Curie (Paris 6) Ph.D. program "Optimisation, jeux et modélisation en économie."

He was a visiting professor at other institutions, including the Ecole Polytechnique Fédérale de Lausanne (1972-1973), the Université Scientifique et Médicale de Grenoble (1974-1975), the Universidad de Chile in Santiago (1994), and INSEAD in Fontainebleau (1997-1998).

Balinski was the founding Editor-in-Chief of the journal *Mathematical Programming* in 1971, one of the founders of the Mathematical Optimization Society in 1970, and President of that society from1986 to 1989.

Research contributions

Balinski's Ph.D. thesis concerned the vertex enumeration problem, the algorithmic problem of listing all vertices of a convex polytope or finding all optimal solutions of a linear program, and some of his subsequent work continued to concern polyhedral combinatorics. The thesis includes the fundamental theorem, published in 1961, that the skeletons of polytopes in n-space viewed as graphs are n-connected, meaning that at least n edges must be removed to disconnect the graph of the remaining vertices and edges. He also proved the Hirsch conjecture for several different classes of polytopes associated with the transportation problem, showed that the diameter of the skeleton of the assignment polytope viewed as a graph is 2, and found the polytope whose vertices are the stable matchings of the university admissions problem.

His contributions to linear and nonlinear optimization include a primal/dual simplex method that incorporates a natural proof of termination and leads to a self-contained, elementary but rigorous, constructive account of the theory and the basic computational tool of linear programming; the use and economic interpretation of dual prices; and a proof that prices in von Neumann's model of an expanding economy are marginal values. His work in integer programming includes the formulation and analysis of the fixed cost transportation problem; one of the first computationally successful practical uses of Gomory's cutting plane algorithm (1968, truck deliveries with cost functions in part concave, in part convex); and an extensive survey paper on integer programming which was awarded INFORM's Lanchester Prize in 1965. In 1970 he published one of the earliest papers on the closure problem and its applications to transportation planning.

Balinski has made important contributions to the theory of electoral decisions, namely, representation and apportionment on the one hand, and voting on the other. His 1982 book with H. P. Young has had direct practical application in apportioning the seats of assemblies to regions in several countries (including the UK). He conceived and developed "biproportional apportionment" that has been adopted (as of 2014) in five of Switzerland's cantonal elections. His 2010 book with Rida Laraki proposes a new theory and method of voting called "majority judgment" where voters evaluate the merit of each candidate in a well-defined ordinal scale (instead of voting for one or several candidates, or rank-ordering them) and majorities determine society's evaluation of each candidate and thereby its rank-ordering of them all. This, they prove, overcomes the most important drawbacks of the traditional theory of voting (including Arrow's impossibility theorem).

Distinctions

Phi Beta Kappa, Williams College, 1954; Lanchester Prize, INFORMS, 1965; I.B.M. World Trade Corporation Fellow, 1969-1970; Lester R. Ford Award, Mathematical Association of America, 1976; Honorary Master of Arts, Privatum, Yale University, 1978; Special Service Award, Mathematical Optimization Society, 1982; Honorary Doctorate (Ehrendoktors, Mathematisch-Naturwissenshaftlichen) Universität Augsburg, 2004; Murat Sertel Lecturer (inaugural lecture), 8th International Meeting of the Society for Social Choice and Welfare, Istanbul, July 2006; Messenger Lecturer, Cornell University, September 2007; IFORS Distinguished Lecturer, INFORMS National Meeting, Washington, D.C., October 2008; Lester R. Ford Award, Mathematical Association of America, 2009; Celebration of Michel Balinski's 78 years at the 23rd International Conference on Game Theory, SUNY Stony Brook, July 2012; John von Neumann Theory Prize, INFORMS, 2013; INFORMS Fellow, 2014.

Publications

Author or co-author of over a hundred articles, Michel Balinski has published three books and edited seven.

Books:

M. L. Balinski and H. P. Young, *Fair Representation: Meeting the Ideal of One Man, One Vote*. Yale University Press, New Haven, CT and London, 1982. 2nd edition, Brookings Institution Press, Washington, D.C., 2001. Japanese translation, Chikura-Shobo Publishing Co., Tokyo, 1987. [Given the George H. Hallet Award, 2008, "[For] a book published at least 10 years ago that has made a lasting contribution to ... representation and electoral systems."]

M. L. Balinski, *Le suffrage universel inachevé*, Editions Belin, Paris, 2004.

M. L. Balinski and Rida Laraki, *Majority Judgment: Measuring, Ranking and Electing*. M.I.T. Press, Cambridge, MA, 2010.

Selected papers:

M. L. Balinski, An algorithm for finding all vertices of convex polyhedral sets. *Journal of the Society for Industrial and Applied Mathematics* 9 (1961) 72-88.

M. L. Balinski, On the graph structure of convex polyhedra in n-space. *Pacific Journal of Mathematics* 11 (1961) 431-434.

M. L. Balinski, Integer programming : methods, uses, computation. *Management Science* 12 (1965) 253-313. Reprinted in G. B. Dantzig and A. F. Veinott, Jr., eds., *Mathematics of the Decision Sciences*, Lectures in Applied Mathematics, American Mathematical Society, vol. 11, 1968, 179-256 ; in H. W. Kuhn, ed., *Proceedings of the Princeton Symposium on Mathematical Programming*, Princeton University Press, 1970, 199-266; and in M. Jünger et al, eds., *50 Years of Integer Programming 1958-2008*, Springer Verlag, 2010, 133-197. [Given the 1965 Lanchester Prize].

M. L. Balinski and W. J. Baumol, The dual in nonlinear programming and its economic interpretation. *The Review of Economic Studies* XXXV (1968) 237-256.

M. L. Balinski and A. W. Tucker, Duality theory of linear programs: a constructive approach with applications. *SIAM Review* 11 (1969) 347-377.

M.L. Balinski, On a selection problem. *Management Science* 17 (1970) 230-231.

M. L. Balinski and H. P. Young, Interpreting von Neumann model prices as marginal values. *Journal of Economic Theory* 9 (1974) 449-463.

M. L. Balinski and H. P. Young, The quota method of apportionment. *American Mathematical Monthly* 82 (1975) 701-730. [Given the Lester R. Ford Award].

M. L. Balinski and G. Demange, An axiomatic approach to proportionality between matrices. *Mathematics of Operations Research* 14 (1989) 700-719.

M. L. Balinski and T. Sönmez, A tale of two mechanisms: student placement. *Journal of Economic Theory* 84 (1999) 73-94.

M. Baïou and M. L. Balinski, The stable allocation (or ordinal transportation) problem. *Mathematics of Operations Research* 27 (2002) 662-680.

M. L. Balinski, What is just? *American Mathematical Monthly* 112 (2005) 502-511.

M. L. Balinski, Fair majority voting (or how to eliminate gerrymandering). *American Mathematical Monthly* 115 (2008) 97-113. [Given the Lester R. Ford Award].

M. L. Balinski, Projets électoraux: le droit rencontre les mathématiques. *Recueil Dalloz* no. 3, 2009, pp. 183-186.

M. L. Balinski and R. Laraki, Jugement majoritaire versus vote majoritaire (via les présidentielles 2011-2012). *Revue Française d'Economie* XXVII (2013) 11- 44.

M. L. Balinski and R. Laraki, Judge: Don't Vote ! *Operations Research* 62 (2014) 483-511.

External links

https://www.google.fr/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&cad=rja&uact=8&ved=0CCQQFjAA&url=https%3A%2F%2Fopus.bibliothek.uni-augsburg.de%2Fopus4%2Ffiles%2F55%2Fbalinski.pdf&ei=hpaCVYOyLciqUdK8gvAE&usg=AFQjCNHwv3Qt5y4-pkbPuPnfWIyDv4mpdg

https://www.informs.org/Recognize-Excellence/Award-Recipients/Michel-L-Balinski

http://www.eurekalert.org/pub_releases/2013-10/ifor-mbo100813.php

http://gtcenter.org/Archive/2012/BalinskiSchedule.html

http://en.wikipedia.org/wiki/Balinski's_theorem

http://www.math.cornell.edu/~billera/misc/Abstracts.html

http://www.orie.cornell.edu/news/index.cfm?news_id=62115&news_back=news_archive%26y%3D2007

http://www.ams.org/samplings/feature-column/fcarc-apportionii3

http://www.college-de-france.fr/site/pierre-rosanvallon/seminar-2012-02-29-10h00.htm

http://mitpress.mit.edu/books/majority-judgment

http://books.google.fr/books?id=hplkLvGkuzwC&pg=PA286&lpg=PA286&dq=michel+balinski&source=bl&ots=nG6tS0j6mn&sig=rg9guUckZnILVz40_VfateABlDA&hl=fr&sa=X&ei=BQVNVI3MH8iBPafEgZgN&ved=0CDUQ6AEwAzgy#v=onepage&q=michel%20balinski&f=false

MICHEL L. BALINSKI

Born: Geneva, Switzerland. United States citizen.

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EDUCATION

1950-1954	B.A. with Highest Honors, Mathematics, Williams College, Williamstown, Massachusetts
1954-1956	M.S., Economics, Massachusetts Institute of Technology, Cambridge, Massachusetts
1956-1959	Ph.D., Mathematics, Princeton University, Princeton, New Jersey

POSITIONS

1954-1956	Research Assistant, Operations Research, Massachusetts Institute of Technology, Cambridge, Massachusetts
1955	Consultant, Borden Mills, Inc., Kingsport, Tennessee and New York, New York
1956-1959	Research Assistant and Assistant in Instruction, Mathematics, Princeton University, Princeton, New Jersey
1959-1961	Mathematician, Mathematica, Inc., Princeton, New Jersey
1960-1963	Research Associate and Lecturer, Mathematics, Princeton University, Princeton, New Jersey
1960-1963	Consultant, RAND Corporation, Santa Monica, California
1962-1969	Lecturer, University of Michigan Summer Engineering Conferences, Ann Arbor, Michigan

1962-1974	Senior Consultant, Mathematica, Inc., Princeton, New Jersey
1963-1965	Associate Professor, Economics and Wharton School, University of Pennsylvania, Philadelphia, Pennsylvania
1965-1969	Associate Professor, Mathematics, Graduate School and University Center, City University of New York, New York
1966-1969	Consultant, Mobil Oil Research Laboratories, Princeton, New Jersey
1969-1977	Professor, Mathematics, Graduate School and University Center, City University of New York, New York
1969-1970	Visiting Fellow, I.B.M. France, Paris
1971	Scientific Consultant, O.R.T.F. (Office de Radiodiffusion-Télévision Française), Paris
1972-1973	Professeur invité, Département de Mathématiques, Ecole Polytechnique Fédérale de Lausanne, Lausanne, Suisse
1974-1975	Professeur associé (visiting Professor), Mathématiques Appliquées-Informatique, Université Scientifique et Médicale de Grenoble, Grenoble
1975-1977	Chairman, System and Decision Sciences, International Institute for Applied Systems Analysis, Laxenburg, Austria
1978-1980	Professor of Organization and Management and of Administrative Sciences, Yale University, New Haven, Connecticut
1978-1980	Senior Consultant, Econ, Inc., Princeton, New Jersey
1980-1982	Professeur, Institut Auguste Comte (to 1981) and Ecole Polytechnique, Paris
1983-1999	Directeur de Recherche (de classe exceptionnelle), C.N.R.S., Laboratoire d'Econométrie de l'Ecole Polytechnique, Paris
1983-1990	Leading Professor of Applied Mathematics and Statistics, and of Economics, State University of New York, Stony Brook, New York
1986-1989	Founder and Director, Institute for Decision Sciences, State University of New York, Stony Brook, New York
1989-1999	Directeur, Laboratoire d'Econométrie de l'Ecole Polytechnique, Paris
1989-1994	Co-founder and co-Director, D.E.A. et Formation Doctorale (Ph.D. program), *Modélisation et méthodes mathématiques en économie : optimisation et analyse stratégique*, Université de Paris 1 and Ecole Polytechnique.
1994	Visiting Professor, Departamento de Ingenieria Matematica, Universidad de Chile, Santiago, Chile

1995-1998	Co-founder and Co-Director, D.E.A. et Formation Doctorale (Ph.D. program), *Optimisation, jeux et modélisation en économie*, Université de Paris 6 (Pierre et Marie Curie), Université de Paris 10 (Nanterre) and Ecole Polytechnique.
1997-1998	Visiting Professor, Technology Management area, INSEAD, Fontainebleau, France.
1999-	Directeur de Recherche (de classe exceptionnelle) émérite, C.N.R.S., Laboratoire d'Econométrie de l'Ecole Polytechnique, Paris

DISTINCTIONS

Phi Beta Kappa 1954

Lanchester Prize of the Operations Research Society of America (INFORMS), 1965

I.B.M. World Trade Corporation Fellow, 1969-1970

Lester R. Ford Award of the Mathematical Association of America, 1975

Honorary Master of Arts, *Privatim*, Yale University, 1978

Special Service Award of the Mathematical Programming Society, 1982

President, Mathematical Optimization Society (formerly Mathematical Programming Society), 1986-1989

Honorary Doctorate, Mathematics (Ehrendoktors, Mathematisch-Naturwissenschaftlichen) , Universität Augsburg, Germany, 2004

Murat Sertel Lecturer, inaugural lecture of 8th International Meeting of the Society for Social Choice and Welfare, Istanbul, July 2006

Messenger Lecturer, Cornell University, September 2007

IFORS Distinguished Lecturer, INFORMS, Washington, D.C., October 2008

George H. Hallett Award of the American Political Science Association, 2008

Lester R. Ford Award of the Mathematical Association of America, 2009

Celebration of Michel Balinski's 78 years, July 17-18 of the 23rd International Conference on Game Theory, Stony Brook, N.Y., July 15-19, 2012.

John von Neumann Theory Prize of INFORMS, 2013

Elected INFORMS Fellow, 2014

PROFESSIONAL ACTIVITIES

INFORMS (formerly, Operations Research Society of America) 1962-

Member, Lanchester Prize Committee, 1974

Mathematical Programming Society 1970-

Member of Founding Committee

Member, Dantzig Prize Committee, 1984-1985

President, 1986-1989.

Associate Editor, *Journal of the Society for Industrial and Applied Mathematics*, 1968-1970

Department Editor, *Management Science*, 1968-1970

Editor-in-Chief (and Founder), *Mathematical Programming,* 1970-1980

Editor-in-Chief (and Founder), *Mathematical Programming Studies*, 1974-1980

Member, Operations Research Council, Advisory Board to the Mayor of the City of New York, 1968-1970

Member, Comité de Direction, Laboratoire d'Automatique et d'Analyse des Systèmes, Toulouse, 1976-1979.

Member, Advisory Committee on I.I.A.S.A., National Academy of Sciences, 1979-1982

Member, Advisory Board, *Journal of Conflict Resolution*, 1991 - 2005

Member, Editorial Board, *Mathematical Social Sciences*, 1993 - 1999

CONSULTING ACTIVITIES

Consultant, Project Director or Principal Investigator for numerous applied mathematics, economics, policy analysis, operations research and management science projects undertaken for the public and private sectors in the United States, France, Italy, Canada , the United Kingdom, . . .

Among these are :

- Long term investment planning, inventory management and resource allocation for the oil industry.
- Statistical quality control for the textile industry.
- Long term planning with particular emphasis on the role of computerization and the value of information for production, for the aeronautical industry.
- Scientific consultant for the television program "Technologie du futur" shown on the "Vingtième siècle" series of TF 1 in 1971.

- Research and consulting for various industries including packaging, natural gas, food, heavy manufacturing, glass and chemical.
- Principal investigator of fundamental research projects in mathematics:

  "On a class of nonlinear optimization problems," Army Research Office (1962-1965);

  "Exploratory research in mathematics," Army Research Office (1965-1974);

  "Axiomatization and optimization for apportionment and social objective functions," National Science Foundation (1975-1981) ;

  "On Monge-Kantorovich problems," National Science Foundation (1989-1990).

- The development of pricing policies for agricultural commodities.
- Application of the theory of games to problems of national defense.
- Evaluation of large scale simulation and gaming programs of international affairs.
- The development of inventory policies for the United States Navy.
- The determination of electoral districts meeting standards of one person, one vote.
- The development of approaches to the allocation of research budgets.
- Devising equitable and economically efficient mechanisms for allotting take-off and landing rights (slots) at congested airports.
- Determining optimal postal distribution networks.
- Evaluating and simulating the effects of different apportionment methods for the Congressional Research Office.
- Reforming the French electoral system.
- Predicting hourly flows in the Paris sewage system as a function of rainfall.
- Expert witness, Conseil constitutionnel (France's supreme constitutional court).
- Advisor on United Kingdom's 2010 *Parliamentary Voting System and Constituencies Bill*.

COLLABORATORS (articles, books and edited books)

Agar Adamson	Yacine Aït-Sahalia	Mourad Baïou	William J. Baumol
Frédéric Bon	Richard Cobb	Richard W. Cottle	Gabrielle Demange
David Gale	Ralph E. Gomory	Jaime Gonzalez	Eli Hellerman
Alan Hoffman	Andrew Jennings	Ron Johnston	Klaus Knorr
Rida Laraki	Claude Lemarechal	Thomas Liebling	Laurent Mann
Iain McClean	Oskar Morgenstern	A.-E. Nobbs	Guillermo Owen
Friedrich Pukelsheim	Richard E. Quandt	Svetlozar T. Rachev	Victoriano Ramirez
Guillaume Ratier	Fred J. Rispoli	Andrew Russakoff	Francis Sand
Niousha Shahidi	Martin Shubik	Tayfun Sönmez	Sylvain Sorin
Kurt Spielburg	Albert W. Tucker	Philip Wolfe	H. Peyton Young

MICHEL L. BALINSKI

BOOKS

1. *Fair Representation : Meeting the Ideal of One Man, One Vote*, Yale University Press, New Haven, Ct. and London, 1982 (with H. P. Young). Second edition, Brookings Institution Press, Washington, D.C., 2001. Japanese translation, Chikura-Shobo Publishing Company, Tokyo, 1987. Given the George H. Hallett Award, 2008 ("[For] a book published at least 10 years ago that has made a lasting contribution to … representation and electoral systems.")

2. *Le suffrage universel inachevé*, Éditions Belin, Paris, 2004.

3. *Majority Judgment: Measuring, Ranking, and Electing*, M.I.T. Press, Cambridge, MA, 2011 (with Rida Laraki).

BOOKS EDITED

1. *Pivoting and Extensions: In Honor of A. W. Tucker,* North-Holland Publishing Company, Amsterdam, 1974.

2. *Approaches to Integer Programming*, North-Holland Publishing Company, Amsterdam, 1975.

3. *Nondifferentiable Optimization***,** North-Holland Publishing Company, Amsterdam, 1975 (with Philip Wolfe).

4. *Computational Practice in Mathematical Programming*, North-Holland Publishing Company, Amsterdam, 1975 (with Eli Hellerman).

5. *Complementarity and Fixed Point Problems*, North-Holland Publishing Company, Amsterdam, 1978 (with Richard W. Cottle).

6. *Polyhedral Combinatorics : Dedicated to the Memory of D. R. Fulkerson,* North-Holland Publishing Company, Amsterdam, 1978 (with Alan J. Hoffman).

7. *Mathematical Programming in Use*, North-Holland Publishing Company, Amsterdam, 1978 (with Claude Lemarechal).

PUBLISHED ARTICLES AND SECLECTED UNPUBLISHED REPORTS

1. "The effect of a traffic light on traffic flow," Operations Research Report, Massachusetts Institute of Technology, 1955 (with John Hess).

2. "The revised simplex method," Master of Science Thesis, Massachusetts Institute of Technology, 1956.

3. *Navy Supply Systems Research Study (NSSRS)*, editor, report prepared for the Bureau of Supplies and Accounts, United States Navy, under contract with Mathematica, Inc., 1960.

4. "Integrated distribution systems," in *NSSRS,* 1960.

5. "Half transportation problems," in *NSSRS*, 1960.

6. "An algorithm for finding all vertices of convex polyhedral sets," *Journal of the Society for Industrial and Applied Mathematics* 9 (1961) 72-88.

7. "On the graph structure of convex polyhedra in n-space," *Pacific Journal of Mathematics* 11 (1961) 431-434.

8. "Fixed cost transportation problems," *Naval Research Logistics Quarterly* 8 (1961) 41-54.

9. "R.S. M.I.A.S. - Fortran mathematical programming system 1 - all solutions," report and computer program prepared at RAND Corporation and released in SHARE Library, 1961.

10. "Economics of the evaluation of unprocessed cottons," report prepared for the Department of Agriculture under contract with Mathematica, Inc., 1961 (with William J. Baumol).

11. "On solving discrete stochastic decision problems," report prepared for the Bureau of Supplies and Accounts, United States Navy, under contract with Mathematica, Inc., 1962.

12. "Pivot transformations, dual linear programs, and simplex methods," Lecture Notes, Princeton University, 1962 (see Marshall Hall, *Combinatorial Theory*, Blaisdell Publishing Co., 1967, chapter 8).

13. "A mutual primal-dual simplex method," in *Recent Methods in Mathematical Programming,* McGraw-Hill, New York, 1963, 17-26 (with R. E. Gomory).

14. "The assignment and transportation problems of linear programming," Lecture Notes, University of Michigan Summer Engineering Conferences, 1963.

15. "On Bender's decompostion and a plant location problem," report prepared for the Army Research Office under contract with Mathematica, Inc., 1963 (with Philip Wolfe).

16. "On an integer program for a delivery problem," *Operations Research* 12 (1964) 300-304 (with Richard Quandt).

17. "A primal method for the assignment and transportation problems," *Management Science* 10 (1964) 578-593 (with R. E. Gomory).

18. "A study of optimal patrol and transit strategies in rectangular barrier zones using mathematical games," report to the Office of Naval Research under contract with Mathematica, Inc., 1965 (with G. Owen and F. Sand).

19. "Integer programming : methods, uses, computation," *Management Science* 12 (1965) 253-313. Reprinted in George B. Dantzig and Arthur F. Veinott, Jr., editors, *Mathematics of the Decision Sciences*, Lectures in Applied Mathematics, American Mathematical Society, vol. 11, 1968, 179-256 ; in H. W. Kuhn, editor, *Proceedings of the Princeton Symposium on Mathematical Programming*, Princeton University Press, 1970, 199-266; and (with an explanatory introduction) in M. Jünger et al, editors, *50 Years of Integer Programming 1958-2008*, Springer Verlag, 2010, 133-197. [Given the 1965 Lanchester Prize of the Operations Research Society of America (INFORMS).]

20. "On finding integer solutions to linear programs," in *Proceedings of the I.B.M. Scientific Computing Symposium on Combinatorial Problems*, 1966, 225-248.

21. "On solving large structured programs," in *Proceedings of the I.F.I.P. Conference, 1965*, New York, 1966, 24-30 (with Richard Cobb).

22. "Some general methods in integer programming," in J. Abadie, editor, *Non-Linear Programming: A Course*, North-Holland Publishing Company, Amsterdam, 1966, 223-247.

23. "An application of utility theory to arms control and disarmament," report prepared for the Arms Control and Disarmament Agency under contract with Mathematica, Inc., 1966 (with F. Sand).

24. "Review of the TEMPER model," report prepared for the Defense Communications Agency under contract with Mathematica, Inc. 1966 (with K. Knorr, O. Morgenstern, F. Sand and M. Shubik).

25. "Notes on a constructive approach to linear programming," in George B. Dantzig and Arthur F. Veinott, Jr., editors, *Mathematics of the Decision Sciences*, Lectures in Applied Mathematics, American Mathematical Society, vol. 11, Providence, R.I., 1968, 38-64.

26. "The dual in nonlinear programming and its economic interpretation," *The Review of Economic Studies* XXXV (1968) 237-256 (with W. J. Baumol).

27. "Labelling to obtain a maximum matching," in R. C. Bose and T. A. Dowling, editors, *Combinatorial Mathematics and its Applications*, University of North Carolina Press, Chapel Hill, 1969, 585-602.

28. "Methods of integer programming : algebraic, combinatorial, and enumerative," in J. Aronovsky, editor, *Progress in Operations Research*, vol. III, John Wiley, New York, 1969, 195-292 (with K. Spielburg).

29. "Airport congestion: the need for an economic allocation," report prepared for the Air Transport Association of America under contract with Mathematica, Inc., 1969.

30. "Various approaches to the districting problem," report prepared for the New Jersey Apportionment Commission under contract with Mathematica, Inc., 1969.

31. "Generation of New Jersey state assembly districts," report prepared for the New Jersey Apportionment Commission under contract with Mathematica, Inc., 1969 (with co-authors).

32. "Duality theory of linear programs: a constructive approach with applications," *SIAM Review* 11 (1969) 347-377 (with A. W. Tucker).

33. "Election of Presidents," Letter to the Editor, *The New York Times*, Sunday, 6 April 1969.

34. "Maximum matching, minimum covering and their connections," in H. W. Kuhn, editor, *Proceedings of the Princeton Symposium on Mathematical Programming*, Princeton University Press, 1970, 303-311.

35. "Recent progress in integer programming," in H. W. Kuhn, editor, *Proceedings of the Princeton Symposium on Mathematical Programming*, Princeton University Press, 1970, 267-302.

36. "On a selection problem," *Management Science* 17 (1970) 230-231.

37. "On perfect matchings," *SIAM Review* 12 (1970) 570-572.

38. Review of R. V. Gamekelidge, editor, *Probability Theory, Mathematical Statistics, and Theoretical Cybernetics,* in *Mathematics of Computation* 25 (1971) 628-629.

39. "Establishing the matching polytope," *Journal of Combinatorial Theory* (Series B) 13 (1972) 1-13.

40. "A note on harmonious families," *Mathematical Programming* 3 (1972) 117-123.

41. "Some properties of the assignment polytope," *Mathematical Programming* 3 (1972) 257-258 (with A. Russakoff).

42. "On the assignment polytope," *SIAM Review* 16 (1974) 516-525 (with A. Russakoff).

43. "Interpreting von Neumann model prices as marginal values," *Journal of Economic Theory* 9 (1974) 449-463 (with H. P. Young).

44. "On two classes of transportation polytopes," in M. L. Balinski, editor, *Pivoting and Extensions*, North-Holland Publishing Company, Amsterdam, 1974, 43-58.

45. "A new method for Congressional apportionment," *Proceedings of the National Academy of Sciences U.S.A.* 71 (1974) 4602-4606 (with H. P. Young).

46. "Le découpage électoral : I. Les inégalités ; II. La distribution optimale," *Le Monde*, 27 and 28 décembre 1974 (with Frédéric Bon).

47. "The quota method of apportionment," *American Mathematical Monthly* 82 (1975) 701-730 (with H. P. Young). [Given the Lester R. Ford Award of the Mathematical Association of America.]

48. "The system and decision sciences area," *I.I.A.S.A. Conference '76*, vol. II, 1976, 177-181.

49. "Apportionment schemes and the quota method," *American Mathematical Monthly* 84, (1977) 450-455 (with H. P. Young).

50. "Addendum to: Apportionment schemes and the quota method," *American Mathematical Monthly* 84 (1977) 542 (with H. P. Young).

51. "On Huntington methods of apportionment," *SIAM Journal on Applied Mathematics* 33 (1977) 607-618 (with H. P. Young).

52. "Fair division and representation," *Options*, autumn 1977, 2-3 (with H. P. Young).

53. "The Jefferson method of apportionment," *SIAM Review* 20 (1978) 278-284 (with H. P. Young).

54. "Stability, coalitions and schisms in proportional representation systems," *American Political Science Review* 72 (1978) 848-858 (with H. P. Young).

55. "Criteria for proportional representation," *Operations Research* 27 (1979) 80-95 (with H. P. Young).

56. "Quotatone apportionment methods," *Mathematics of Operations Research* 4 (1979) 31-38 (with H. P. Young).

57. "Reply," *The American Political Science Review* 73 (1979) 552-553 (with H. P. Young).

58. "Fair congressional apportionment," *Proceedings of the International Conference on Cybernetics and Society*, I.E.E.E. Systems, Man and Cybernetics Society, Denver, Colorado, 1979, 217.

59. "The Webster method of apportionment," *Proceedings of the National Academy of Sciences U.S.A.* 77 (1980) 1-4 (with H. P. Young).

60. "The allocation of runway slots by auction," report prepared for Office of Aviation Policy, Federal Aviation Administration, U. S. Department of Transportation, under contract with Econ, Inc., April 1980. Report No. FAA-AVP-80-3, in 3 volumes (with F. Sand).

61. "When House seats shift, fractions count," *Washington Post*, Outlook Section, Sunday, November 30, 1980, D5 (with H. P. Young).

62. "The challenge of applied problems to theory development," *I.I.A.S.A. Reports : A Journal of International Applied Systems Analysis* 3 (1981) 193-194.

63. "Parliamentary representation and the amalgam method," *Canadian Journal of Political Science* XIV (1981) 797-812 (with H. P. Young).

64. "Fair representation in the European Parliament," *Journal of Common Market Studies* XX (1982) 361-373 (with H. P. Young).

65. "Signatures des points extrêmes du polyèdre dual du problème de transport," *Comptes Rendus de l'Académie des Sciences Paris*, 296 (1983) série I, 457-459.

66. "Apportioning the United States House of Representatives," *Interfaces* 13.4 (1983) 35-43 (with H. P. Young).

67. "Fair electoral distribution," *Policy Options/Options Politiques* 4 July/August 1983, 30-32 (with H. P. Young).

68. "The Hirsch conjecture for dual transportation polyhedra," *Mathematics of Operations Research* 9 (1984) 629-633.

69. "Faces of dual transportation polyhedra," in B. Korte and K. Ritter, editors, *Mathematical Programming at Oberwolfach II*, Mathematical Programming Study 22, 1984, 1-8 (with A. Russakoff).

70. Review of S. J. Brams and P. C. Fishburn, *Approval Voting*, in *Optima*, January 1984.

71. "Evaluation of apportionment methods," report prepared for the Congressional Research Service, United States Library of Congress, September 1984 (with H. P. Young).

72. "Signature methods for the assignment problem," *Operations Research* 33 (1985) 527-536.

73. "Auctioning landing rights at congested airports," in H. P. Young, editor, *Cost Allocation: Methods, Principles, Applications,* North-Holland Publishing Company, Amsterdam, 1985, 179-192 (with F. M. Sand).

74. "The apportionment of representation," in H. P. Young, editor, *Fair Allocation,* Proceedings of Symposia in Applied Mathematics 33, American Mathematical Society, Providence, R.I., 1985, 1-29 (with H. P. Young).

75. "How to make representation equal," *Policy Options/Options Politiques* 6, November 1985, 10-14 (with Agar Adamson).

76. "A competitive (dual) simplex method for the assignment problem," *Mathematical Programming* 34 (1986) 125-141.

77. "A good dual simplex method for the assignment problem," *Journées du 20ième Anniversaire du Groupe Combinatoire de l'AFCET*, 3-5 décembre 1986, 197-225.

78. "On the average length of lexicographic paths," *Mathematical Programming* 35 (1986) 362-364 (with Th.-M. Liebling and A.-E. Nobs).

79. "Notes on defining proportionality between matrices," in *Proceedings of the Eighth Mathematical Programming Symposium, Japan*, Chugoku Electric Power Co., Hiroshima, 5-6 November 1987, 1-17 (with Gabrielle Demange).

80. "Le redressement des tables de contingence : deux nouvelles approches," in *Mélanges économiques : Essais en l'honneur de Edmond Malinvaud*, Economica, Paris, 1988, 897-914 (with Y. Aït-Sahalia and Gabrielle Demange).

81. "Presidential address," 13th Symposium on Mathematical Programming, Tokyo, Japan, August 1988, *Optima* 25, 1988.

82. "An axiomatic approach to proportionality between matrices," *Mathematics of Operations Research* 14 (1989) 700-719 (with Gabrielle Demange).

83. "Algorithms for proportional matrices in reals and integers," *Mathematical Programming* 45 (1989) 193-210 (with Gabrielle Demange).

84. "On the core of the assignment game," in Lev J. Leifman, editor, *Functional Analysis, Optimization, and Mathematical Economics: A Collection of Papers Dedicated to the Memory of Leonid Vital'evich Kantorovich,* Oxford University Press, New York and Oxford, 1990, 274-289 (with David Gale).

85. "Rounding proportions," Technical Report No. 127, Program in Statistics and Applied Probability, University of California, Santa Barbara, April 1990 (with S. T. Rachev).

86. “On the core of the assignment game (abstract),” in T. Ichiishi, A. Neyman and Y. Tauman, editors, *Game Theory and Applications*, Academic Press, Inc., New York, 1990, 373-374 (with David Gale).

87. “The adjustment of contingency tables : two new approaches,” in Paul Champsaur *et al*, editors, *Essays in Honor of Edmond Malinvaud* (vol. 3), The MIT Press, Cambridge, Mass. and London, 1990, 99-114, English translation of #80 (with Y. Aït-Sahalia and Gabrielle Demange).

88. “Maximum matchings in bipartite graphs via strong spanning trees,” *Networks* 21 (1991) 165-179 (with J. Gonzalez).

89. “Gaspard Monge : Pour la patrie, les sciences et la gloire,” in C. Carasso, C. Conca, R. Correa, and J.-P. Puel, editors, *Applied Mathematics for Engineering Sciences - Mathématiques Appliquées aux Science de l’Ingénieur - Matématicas Aplicadas a la Ciencas del Ingeniero*, 2ième colloque franco-chilien de mathématiques appliquées, Cépaduès-Editions, Toulouse, 1991, 21-37.

90. “Mathematical programming : journal, society, recollections,” in J.-K. Lenstra, A. H. G. Rinnooy Kan and A. Schrijver, editors, *History of Mathematical Programming: a Collection of Personal Reminiscences*, CWI and North-Holland Publishing Company, Amsterdam, 1991, 5-18.

91. “Rounding proportions: rules of rounding,” *Numerical Functional Analysis and Optimization* 14 (1993) 475-501 (with S. T. Rachev).

92. “The problem with apportionment,” *Journal of the Operations Research Society of Japan* 36 (1993) 134 - 148.

93. “Optimisation,” *Revue Scientifique et Technique de la Défense*, La recherche à l'École Polytechnique, numéro spécial bicentenaire de l'École (1993) 173-179.

94. “Signature classes of transportation polytopes,” *Mathematical Programming* 60 (1993) 127-144 (with Fred J. Rispoli).

95. “Le vote par assentiment : un k-system de vote sincère,” Cahier du Laboratoire d’Économétrie de l’École Polytechnique, n° 386, janvier 1993 (with Laurent Mann).

96. “Le choix d'une méthode de répartition de sièges,” in *Rapport sur le problème de la réforme du mode de scrutin pour l'élection des députés*, Commission de Réforme du Mode de Scrutin, février, 1993, 56 - 65.

97. “Si le mode de scrutin avait été modifié . . .,” *Le Monde*, p. 12, 3 avril 1993, (with Laurent Mann).

98. "Some developments in the theory of rounding proportions," *Bulletin of the International Statistical Institute*, contributed papers, 49th session, book 1, Firenze 1993, 71 - 72 (with B. D. Athanasopoulos et S. T. Rachev).

99. "Apportionment," in S. M. Pollock, M. H. Rothkopf and A. Barnett, editors, *Operations Research and the Public Sector,* Handbooks in Operations Research and Management Science, Vol. 6, North-Holland, Amsterdam, 1994, 529-560 (with H. P. Young).

100. "How should data be rounded ?," in Ludger Rüschendorf, Berthold Schweizer and Michael D. Taylor, editors, *Distributions with Fixed Marginals and Related Topics,* Lecture Notes -- Monograph Series Vol. 28, Institute of Mathematical Statistics, Hayward, California, 1996, 33-44.

101. "A case study of electoral manipulation : the Mexican laws of 1989 and 1994," *Electoral Studies* 15 (1996) 203-217 (with Victoriano Ramírez González).

102. "El problema del reparto proporcional de escaños," *Boletín de la Sociedad Española de Matemáica Aplicada* abril 1996, 3-34 (with Victoriano Ramírez González).

103. "Los sistemas electorales de 1989-90 y 1994 para la elección de diputados en México: Análisis y crítica," *Estudios Sociológicos* (Mexique) XV (1997) 137-185 (with Victoriano Ramírez González).

104. "Of stable marriages and graphs, and strategy and polytopes," *SIAM Review* 39 (1997) 574-604 (with Guillaume Ratier).

105. "Rounding proportions : methods of rounding," *The Mathematical Scientist* 22 (1997) 1-26 (with Svetlozar T. Rachev).

106. "Mexican electoral law: 1996 version," *Electoral Studies* 16 (1997) 329-340 (with Victoriano Ramírez González).

107. "Graphs and marriages," *American Mathematical Monthly* 105 (1998) 430-445 (with Guillaume Ratier).

108. "A simple approach to the product variation problem *via* axiomatics," *Operations Research Letters* 22 (1998) 129-135 (with Niousha Shahidi).

109. "Mexico's 1997 apportionment defies its electoral law," *Electoral Studies* 18 (1999) 117-124 (with Victoriano Ramírez González).

110. "Parametric methods of apportionment, rounding and production," *Mathematical Social Sciences* 37 (1999) 107-122 (with Victoriano Ramírez).

111. "A tale of two mechanisms: student placement," *Journal of Economic Theory* 84 (1999) 73-94 (with Tayfun Sönmez).

112. "Découpages électoraux et règles mathématiques," *Le Monde*, August 28, 1999.

113. "The stable admissions polytope," *Mathematical Programming* 87 (2000) 427-39 (with Mourad Baïou).

114. "Many-to-many matching: stable polyandrous polygamy (or polygamous polyandry)," *Discrete Applied Mathematics* 101 (2000) 1-12 (with Mourad Baïou).

115. "Modèles mathématiques," in *Dictionnaire du Vote* (directeurs: Pascal Perrineau and Dominique Reynié), Presses Universitaires de France, February 2001, 646-649.

116. "Les enjeux du recrutement," *Pour la science* (October 2001) 64-70.

117. "The stable allocation (or ordinal transportation) problem," *Mathematics of Operations Research* 27 (2002) 485-503; corrected version *Mathematics of Operations Research* 27 (2002) 662-680 (with Mourad Baïou).

118. "Compte-rendu d'une expérience de vote à l'Institut d'Etudes Politiques de Paris," Working paper, Laboratoire d'Économétrie, École Polytechnique, March 2002 (with J.-F. Laslier and K. van der Straeten).

119. "La justice électorale," *Pour la science* (April 2002) 44-45.

120. "Le scrutin," *Pour la science* (April 2002) 46-51. German translation, "Wer wird Präsident ?," *Spektrum der Wissenschaft* (September 2002) 74-79; reprinted in *Fairness, Kooperation, Demokratie: Die Mathematik des Sozialverhaltens,* Dossier 5 (2006), Spektrum der Wissenschaft 6-12.

121. "Répartitions des sièges," *Pour la science* (April 2002) 52- 57.

122. "Une 'dose' de proportionnelle : le système électorale mexicain," *Pour la science* (April 2002) 58-59. German translation, "Verhältniswahlrecht häppchenweise," *Spektrum der Wissenschaft* (October 2002) 72-74; reprinted in *Fairness, Kooperation, Demokratie: Die Mathematik des Sozialverhaltens,* Dossier 5 (2006), Spektrum der Wissenschaft 19-21.

123. "Découpage électoral," *Pour la science* (April 2002) 60-64 (with Mourad Baïou).

124. "Le pouvoir des votes," *Pour la science* (April 2002) 65-69 (with Rida Laraki).

125. "Charcuterie fine," 300ème emission, spécial Elections, *Archimède* (the scientific and technical television program of Arte) April 9, 2002. Author (with Mourad Baïou).

126. "Résultats du vote par assentiment," *Pour la science* (June 2002) (with R. Laraki, J.-F. Laslier and K. van der Straeten).

127. "Claude Berge: Le père de la théorie moderne des graphes, " *Le Monde*, July 13, 2002.

128. "Admissions and Recruitment," *American Mathematical Monthly* 110 (2003) 386-399 (with Mourad Baïou).

129. "Le vote par assentiment : une expérience," Laboratoire d'Économétrie, École Polytechnique, Cahier n° 2003-013, juillet 2003 (with Rida Laraki, Jean-François Laslier and Karine Van der Straeten).

130. "Quelle équité ?" *Pour la science* (septembre 2003) 82-87. German translation, "Die Mathematik der Gerechtigkeit," *Spektrum der Wissenschaft* (March 2004) 90-97 ; reprinted in *Fairness, Kooperation, Demokratie: Die Mathematik des Sozialverhaltens,* Dossier 5 (2006), Spektrum der Wissenschaft 24-31.

131. "Student admissions and faculty recruitment," *Theoretical Computer Science* 322 (2004) 245-265 (with Mourad Baïou).

132. "Le suffrage dit 'universel'," *Le Monde*, 29 octobre 2004, p. 25.

133. "What is just?" *American Mathematical Monthly* 112 (2005) 502-511.

134. "Lois électorales. Le vote à la découpe," *Commentaire* N° 110, été 2005, pp. 439-451.

135. "Le suffrage n'est plus universel," *Libération*, 27 juillet 2005, pp. 30-31.

136. "L'élection présidentielle est-elle démocratique ?" *Le Monde*, 29 novembre 2005, p. 26.

137. "Matrices and politics," in E. P. Liski, J. Isotalo, S. Puntanen and G. P. H. Styan (Eds.), *Festschrift for Tarmo Pukkila*, Department of Mathematics, Statistics and Philosophy, University of Tampere, Finland, 2006, 233-242 (with Friedrich Pukelsheim).

138. "Apportionment : uni- and bi-dimensional," in B. Simeone and F. Pukelsheim (Eds.), *Mathematics and Democracy. Recent Advances in Voting Systems and Collective Choice,* Springer, Berlin and Heidelberg, 2006, 43-53.

139. "The Erice Decalogue," in B. Simeone and F. Pukelsheim (Eds.), *Mathematics and Democracy. Recent Advances in Voting Systems and Collective Choice,* Springer, Berlin and Heidelberg, 2006, xi-xii.

140. "Flots et couplages," *Graphes et applications 1* (directeurs: Jean-Claude Fournier et Michel Las Vergnas), Hermes Science/Lavoisier, Paris, 2007, pp. 183-223 (with Mourad Baïou).

141. "Equitable representation and recruitment," in Kurt Spielberg and Monique Guignard-Spielberg, editors, *History of Integer Programming: Distinguished Personal Notes and Reminiscences* (*Annals of Operations Research*, vol. 149, 2007) 27-36.

142. "Characterizations of the optimal stable allocation mechanism," *Operations Research Letters* 35 (2007) 392-402 (with Mourad Baïou).

143. "A theory of measuring, electing and ranking," *Proceedings of the National Academy of Sciences, USA* 104 (2007) 8720-8725 (with Rida Laraki).

144. "Die Mathematik der deppelten Gerechtigkeit," *Spektrum der Wissenschaft*, April 2007, 76-80 (with Friedrich Pukelsheim).

145. "Le dilemme du vote 'utile', " *Le Monde*, 30 mars 2007, p. 22 (with Rida Laraki).

146. "Le jugement majoritaire : l'expérience d'Orsay," *Commentaire* N° 118, été 2007, pp. 413-419 (with Rida Laraki).

147. "Fair majority voting (or how to eliminate gerrymandering)," *American Mathematical Monthly* 115 (2008) 97-113. [Given the Lester R. Ford Award of the Mathematical Association of America.]

148. "Redistricting in France under changing electoral rules," *in* B. Grofman and L. Handley (eds.), *Redistricting in Comparative Perspective*, Oxford University Press, 2008, pp. 173-190.

149. "Une des racines du mal socialiste," *Libération*, 26 octobre 2008 (with Rida Laraki).

150. "The majority judgement : a new theory for better decisions," *IFORS News*, December 2008, 5-6.

151. "Projets électoraux: le droit rencontre les mathématiques," *Recueil Dalloz* no. 3, 2009, pp. 183-186.

152. "Chausse-trapes du redécoupage électoral," *Le Monde*, 10 février 2009.

153. "Monotonic incompatibility between electing and ranking," *Economics Letters* 105 (2009) 145-147 (with Andrew Jennings and Rida Laraki).

154. "Gale in Paris," *Games and Economic Behavior* 66 (2009) 594-597 (with Rida Laraki and Sylvain Sorin).

155. *Drawing a New Constituency Map for the United Kingdom: The Parliamentary Voting System and Constituencies Bill 2010*, report prepared for the British Academy, Policy Center, British Academy, London, September 2010 (with Ron Johnston, Iain McLean and Peyton Young, and research assistance from Angela Cummine).

156. "The majority judgment: a new mechanism for electing and ranking," in *Beyond the Ballot Box*, Center for Bhutan Studies, Thimphu, Bhutan, 2010, pp. 257-268.

157. "PS : le jugement majoritaire, meilleur système de primaires," *Rue89.* See http://www.rue89.com, August 30, 2010 (with Rida Laraki).

158. "Judge: *Don't Vote!*" *Operations Research* 62 (2014) 483-511 (with Rida Laraki).

159. "Election by majority judgment: experimental evidence," In *In Situ and Laboratory Experiments on Electoral Law Reform: French Presidential Elections*, eds. B. Dolez, B. Grofman, and A. Laurent, Springer, 2011, pp. 13-54 (with Rida Laraki).

160. "Rendre les élections aux électeurs : le jugement majoritaire," Note, Terra Nova, 21 April 2011 (with Rida Laraki). See http://www.tnova.fr/note/rendre-les-lections-aux-lecteurs-le-jugement-majoritaire

161. "Le jugement majoritaire : une nouvelle théorie du vote," Lecture, Collège de France, 29 Febuary 2012. See http://www.college-de-france.fr/site/pierre-rosanvallon/seminaire-du-29-fevrier-2012-michel-balinski-et-rida-laraki-le-jugement-majoritaire-une-nouvelle-theorie-du-vote.htm

162. "Ne votez pas, jugez !" *Pour la science* (April 2012) 22-28 (with Rida Laraki).

163. "Review: *A Mathematical Look at Politics*," by E.A. Robinson Jr., and D. H. Ullman (CRC Press, Boca Raton, FL)," *Siam Review* 54 (2012) 394-395.

164. "Parametric vs. divisor methods of apportionment,"*Annals of Operations Research* 215 (2014) 39-48 (with Victoriano Ramirez).

165. "How best to rank wines," in E. Giraud-Héraud and M.-C. Pichery (eds.), *Wine Economics: Quantitative Studies and Empirical Applications*, 2013. London: Palgrave (with Rida Laraki) 149-172.

166. "Jugement majoritaire versus vote majoritaire (via les présidentielles 2011-2012)," *Revue Française d'Economie* XXVII (2013) 11- 44 (with Rida Laraki).

167. "What *should* 'majority decision' mean?" in Jon Elster and Stéphanie Novak (eds.), *Majority Decisions*, 2014, pp. 103-131. Cambridge, UK: Cambridge University Press (with Rida Laraki).

168. "La justice électorale : la réaliser en Tunisie," L'ATUGE, Tunisia, 2014 (with Rida Laraki). See http://www.leaders.com.tn/article/comment-realiser-la-justice-electorale-en-tunisie?id=13470

169. "A letter regarding Steven J. Brams's review of *Majority Judgment,*" in *American Scientist* (with Rida Laraki). See http://www.americanscientist.org/bookshelf/pub/a-letter-regarding-steven-j-bramss-review-of-majority-judgment

170. “How to make the House of Representatives representative,” *TheConversation*, posted 21 October 2014. See http://theconversation.com/how-to-make-the-house-of-representatives-representative-32921

171. “Majority measures,” Cahier n° 2015-05, Département d’Economie, Ecole Polytechnique, Palaiseau, France.

172. “Primaires: Et si c’était à refaire?” Terra Nova, 25 April 2015 (with Alain Bergounioux, Anne-Lorraine Bujon, Rida Laraki, and Thierry Pech). See http://www.tnova.fr/sites/default/files/25042015%20-%20Primaires,%20et%20si%20c'était%20à%20refaire_0.pdf

173. “Voting on the ox,” Letter, *New York Review of Books*, LXII(4), May 7-20, 2015, p. 58.